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Generalized Fuzzy Digraph

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Abstract

A tool for portraying several real networks is a fuzzy graph. The fuzzy graphs are merely suitable to depict some networks due to various edge limitations. The expected results of research on a variety of applications remain highly outgoing and might be surpassing. In the present article, we analysed the centrality of the community networks utilizing a Generalized Fuzzy Digraph (GFD) to figure out who make use of the Telegram channel vigorously for the preparation of NET examination. Utilizing the cartesian product the three values are turned into one value for fuzzification, leading to the values to get fuzzified. This paper applies GFD to demonstrate an application to identify the central individual in any social group such as Telegram, Facebook, Instagram etc., which provides a best result in many crucial situations.

Keywords: Centrality, Degree Centrality, Generalized Fuzzy Digraph, Membership Value

1. Introduction

Today, all processes, including networks, pathways, schedules, photographs, etc., use the graph theory concept. A social media platform can be compared to a graph, with each account (person, business, etc.) represented by a vertex and each account's connections represented by an edge. A group of individuals or organizations linked together by these ties constitutes a social media. A million individuals utilize smartphones, and they like connecting and exchanging information using social media apps. Social media like Twitter and Facebook have recently grown significantly in daily life. Friends, businesses, and individuals are all part of the social media. In the development of specific tasks within social media is locating a significant key or powerful node. The center of a network is referred to as being central or primary. Determining centrality in a social media thus constitutes an important procedure.

To tackle problems in real life, a wide range of primary projects have been selected and developed daily. Shimbel[1] suggested using an estimation of centrality based on the shortest path to examine a telephone number. For each concept, Freeman[2] developed three different form of centrality measurements: an absolute calculation, a centralization value for the entire device and a comparative centrality value for a network position. These metrics have been evaluated for the small-group experimental culture. Additionally, a data center designed by Zelen and Stephenson[3] was implemented, with a focus on data transmitted via a linked media between two vertices. Brandes[4] developed a more rapid centrality algorithm, reducing the amount of time and space required for comparison analysis. To find out the number of times a vertex appears in each connected network subgraph, Rodriguez-Velazquez and Estrada[5] provided a formula for subgraph centrality. Rodriguez *et al.*[6] established as the walking order increases, the impact on centrality diminishes, assuming that closed walks are appropriately weighed with increased centrality as a focal aspect. Firstly, Freeman[2] highlighting a mathematical model rooted in connections to a vertex within the context of degree power and centrality. Bonacich[7] has supplied a thorough definition of degree centrality. In addition, Weighed network centrality metrics were enhanced by Opsahl *et al.*[8]. Joyce and superiors[9] employed leverage centrality, a recent development, to study the human brain's neural network. For an improved understanding of the rating list, Liu *et al.*[10] offered an upgraded approach. This method analyses the set of k-shell values and the shortest path

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from a media core target node, also known as the k-shell values. The central position of the neighbourhood, as described by Bae and Kim[11], includes a list of every node in the network, its grade, and the coreness of its neighbours. Liu *et al.*[12] proposed a method for achieving a strong local centrality in a variety of networks. Then, Wang *et al.*[13] proposed a weighted neighbourhood centrality that would improve the accuracy of notable node ranking. Samanta *et al.*[14] introduced a novel concept for calculating centrality in a network. Furthermore, Sometimes the state of the system is vague or confusing; in these cases, a fuzzy graph can capture the ambiguity or uncertainty. Kauffman[15] gave the initial explanation of a fuzzy graph. Mahapatra *et al.*[16] describe several applications for the fuzzy graph. The membership values of the edge of these two fuzzy graphs are similar in that they are both less than the minimum of their end membership values of the vertex. Assume that social media is represented by fuzzy graphs. Both social units are identified as fuzzy nodes in this case. Many variables can alter the vertical membership values. Suppose the relationship between these units and the important source is described by fuzzy edges. As a result, information transfer determines the membership advantage. A fuzzy graph cannot adequately express this type of circumstance. The generalized fuzzy graph is discussed by Sarkar and Samanta[17].

Section I deals with preliminary concepts that are utilized to develop this context. Section II discusses about the main concept of this paper, in which Generalized fuzzy digraph is defined and its applications is briefly discussed in section III and section IV, concludes the paper.

2. Preliminaries

Definition 2.1. If V is a collection of object denotes generically by v, then a *fuzzy set* $\overline{A}$ in V is a set of ordered pairs.[18]

$$\widetilde{A} = \{(V, \mu_{\overline{A}}(v)) \mid v \in V\}$$

Where $\mu_{\overline{A}}(v)$ is called the membership function of V.

Definition 2.2. Let $U, V \subseteq R$ be the universal sets, then

$$\widetilde{R} = \{((u,v), \mu_R(u,v)) \mid (u,v) \in U \times V\}$$

is called a *fuzzy relation* on $U \times V$.[18]

Definition 2.3. A *fuzzy graph* $\delta = (V, \lambda, \eta)$, V is the non-empty vertex set and $\lambda : V \to [0,1]$ and $\eta : V \times V \to [0,1]$ such that $\eta(u,v) \le \min\{\lambda(u), \lambda(v)\}$, where $\lambda(u)$ and $\lambda(v)$ are the vertex membership values $u, v \in V$ and $\eta(u,v)$ denotes the edge membership values (η) of δ.[15]

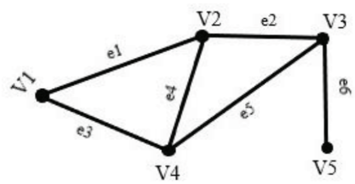
Definition 2.4. Let $\alpha : V \to [0,1]$ and $\beta : V \times V \to [0,1]$ be two functions, where, V is the non-empty vertex set. Suppose $A = \{(\alpha(u), \alpha(v)) \mid \beta(u,v) > 0\}$. Then (V, α, β) is known as *generalized fuzzy graph* if there exists a function $\psi : A \to (0,1]$ such that $\beta(u,v) = \psi(\alpha(u), \alpha(v)) \forall u, v \in V$. Here, $\alpha(u)$, $u \in V$ is the vertex membership value (λ) and $\beta(u,v)$, for all $(u,v) \in v \times V$ is the edge membership value (η).[17]

Table 1. Basic notations

Notation	Meaning
δ	Fuzzy graph
V	Vertex set
E	Edge set
λ	Vertex membership value
η	Edge membership value
Z	Centrality degree of the vertex
Z'	Normalization vertex degree centrality
z	Degree of the vertex
C_i	Vertex centrality by generalized fuzzy digraph
$C(V)$	Centrality of Vertex
$D(V)$	Degree Centrality of vertex

3. Main Results

Definition 3.1. A Directed fuzzy graph ($\overrightarrow{F_G}$) is said to be a *generalized fuzzy digraph* (V, λ, η), if there exists a relationship between $\lambda_D \& \eta_D$ and suppose $A = \{\lambda(u), \lambda(v) \mid \eta(u,v) > 0\}$, then $\phi : A \to (0,1]$ has $\overrightarrow{\eta}_D(u,v) \le \min\{\lambda_D(u), \lambda_D(v)\} \forall u, v \in V$. Here $\lambda(u), \forall u \in V$ denotes the vertex membership value of u and $\eta(u,v) \forall u, v \in V$ denotes the membership value of edge (u,v).



Definition 3.2. The number of nodes that are connected to each other shows the central node of a network, which is calculated through centrality degree. The degree of centrality of the vertex u is indicated by $z(u)$. Then the Normalization of centrality degree is given by,

$$Z'(u) = \frac{z(u)}{(n-1)}$$

where, n is the number vertices in a given network.

Definition 3.3. Let $\vec{G} = (V, E)$ be a directed graph, where V is a finite nonempty set of vertices and $E = \{(u,v) : u, v \in V, u \neq v\}$. A *directed fuzzy graph* $\overrightarrow{F_G} = (\lambda, \eta)$ is a pair of two functions $\lambda : V \to [0,1]$ and $\eta : V \to [0,1]$ such that

$$\eta(u,v) \leq \min\{\lambda(u) \wedge \lambda(v)\}, \forall u, v \in V$$

Theorem 1. *Let $\phi = (V, \lambda, \vec{\eta})$ be a GFD with $|V| = n$ and $C_i \in C(V)$ then $0 \leq C_i \leq 1$.*

Proof. Let ϕ be a generalized fuzzy digraph and $|V| = n$. So, $0 \leq \eta(a,b) \leq 1, \eta(a,b)$ be the membership value of the edge (a,b). Then, $C(V)$ is $C_i = \frac{\sum_{j=1}^{m} \overrightarrow{\eta_j}}{n-1}$ where $i = 1, 2, \ldots, m$ and the number of edges (m) connected with the vertex v_i. So, $0 \leq C_i \leq 1$ is true.

Theorem 2. *Let $\phi = (V, \lambda, \vec{\eta})$ be a GFD with $|V| = n$ and $C_i \in C(V)$ and $Z_i \in D(v_i)$ of underlying crisp graph of ϕ then $C_i \leq Z_i'$.*

Proof. Given, ϕ is a GFD and $C_i \in C(V), \forall v_i = v_1, v_2, \ldots, v_n$ then $C_i = \frac{\sum_{j=1}^{m} \overrightarrow{\eta_j}}{n-1}$ where m is number of edges connected with the vertex v_i, n is the no. of vertex and the value $\vec{\eta}_j$ is $0 \leq \overrightarrow{\eta_j} \leq 1$. $Z_i' \in C(V)$ of underlying crisp graph of ϕ. Then Z_i = degree of the vertex v_i. In the crisp graph, all the edges are considered as 1. However, the greatest value of an edge in a fuzzy is one. So, $C_i \leq Z_i'$ is true.

Theorem 3. *Let $\phi = (V, \lambda, \vec{\eta})$ be a complete GFD with $|V| = n$ and C_i denotes $C(V)$ and $Z_i \in D(V)$ of underlying crisp graph of ϕ then the value of C_i is different for all vertices, but the values of Z_i is equal.*

Proof. Let ϕ be a GFD and $Z_i' \in C(V)$ in Figure 2 underlying crisp graph is represented by v_i underlying crisp graph of ϕ Then Z_i = degree of the vertex v_i. Subsequently in the graph, every vertex has the same degree. Additionally, each edge in the crisp graph is regarded as one. Therefore, the value of Z_i is the same for each of the vertices in a fuzzy graph, but the edge membership value will differ occasionally. Thus, C_i's value might vary for each of the vertices.

4. Centrality Measure for the Generalised Fuzzy Digraph

Definition 4.1. Let $\phi = (V, \lambda, \vec{\eta})$ be a GFD and $|V| = n$ then $C(V)$ is denoted by C_i and defined by

$$C_i = \frac{\sum_{j=1}^{m} \overrightarrow{\eta_j}}{n-1}$$

Where $i = 1, 2, \ldots, n \,\&\, m$ is the no. of edges that are connected to the vertex v_j.

An algorithm to calculate the GFD's Centrality.

A GFD must be provided as an input, and $|V'|$ must equal n.

Output: The GFD's centrality of vertices $C(V)$.

Step 1: This GFD's vertices are defined as $v_1, v_2, \ldots, v_n$.

Step 2: Next, use the formula to determine the vertex centrality " v_1 "

$$C_i = \frac{\sum_{j=1}^{m} \overrightarrow{\eta_j}}{n-1}$$

where $m \in C(v_1)$ and is the number of connections a vertex has with vertex " v_1 "

Step 3: Continue Step 2 until all vertices are considered once.

Step 4: These nodes with the maximum values of centrality of $\{C_1, C_2, \ldots, C_n\}$ are the network's central nodes.

An algorithm to calculate the GFD's Degree Centrality.

A GFD must be provided as an input, and $|V'|$ must equal n.

Output: The GFD's degree centrality of vertices $C(V)$.

Step 1: This GFD's vertices are defined as $v_1, v_2, \ldots, v_n$.

Step 2: Next, figure out the Degree centrality of the vertex "v1" using the formula where $Z(v_1)$ is the vertex's degree and n is total no. of network's vertex. This expression stands for the $D(v_1)$.

Step 3: Repeat Step 2, until all vertices are visited once.

Step 4: The nodes with a maximum Degree Centrality value of $\{C_1, C_2, \ldots, C_n\}$ are the network's central nodes.

Example:

Consider a GFD $\overrightarrow{F_G}$ with $V = \{v_1, v_2, \ldots, v_6\}$ as the vertex set and $E = \{v_1v_2, v_1v_5, v_3v_2, v_4v_2, v_5v_2, v_3v_4, v_4v_5, v_6v_5\}$ be the edge set of the DFG is displayed below:

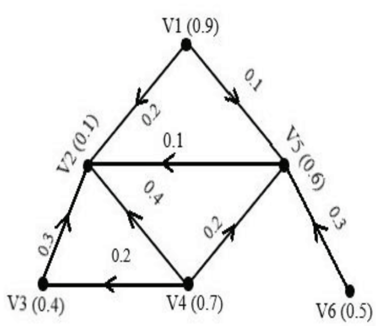


Figure 1 : A directed fuzzy graph

A membership, centrality values and centrality degree of one set of data points provided above. Table 2 exhibits the vertex membership values in the fuzzy directed graph. The subsequent tables manifest the membership values of both vertices and edges and the vertex centrality values respectively.

Table 2. Membership value of vertices of Figure 1

Vertex(V)	Membership value	Centrality
v_1	0.9	0.06
v_2	0.1	0.2
v_3	0.4	0.1
v_4	0.7	0.16
v_5	0.6	0.14
v_6	0.5	0.06

Table 3. Membership value of edges of Figure 1

Edges (E)	Membership value
(v_1, v_2)	0.2
(v_1, v_5)	0.1
(v_3, v_2)	0.3
(v_4, v_2)	0.4
(v_5, v_2)	0.1
(v_3, v_4)	0.2
(v_4, v_5)	0.2
(v_6, v_5)	0.3

5. Applications

In the modern world, online social media platforms are important. Currently, social media are available to everyone. Social networking services provide users with an online platform to establish social networks, connections, or in-person contacts with others who have similar interests, careers, or life experiences. Users of social networking services can communicate with people in their network online by swapping posts, digital photos and videos, and ideas. Nearly 2.13 billion people use Facebook per month, and typically there are 1.4 billion daily active users. In an era, where instant communication is paramount, Telegram has emerged as a dynamic and secure messaging platform that has redefined the way we connect with others. Within the platform, messages, links, and videos can be shared immediately. Your internet connection, the size of the file or video, and the current server load on Telegram are some of the variables that affect how quickly you can share something. In comparison to many other messaging systems, the delivery of text messages and small files often takes a few seconds. Larger files or high-quality films may take a little longer. Telegram might handle quick and effective file sharing hangs to its infrastructure and diverse server network. The values of the videos, links, messages exchanged on Telegram channel are taken onto account in the generalized fuzzy digraph that is displayed below. Edges symbolizes a community with shared messages, links and videos. The user account in one particular telegram channel is shown by the vertices. Depending on how many messages, links, and videos are shared over the last three days, a vertex's membership value is determined. Table 5 displays all the calculations. Table 6 displays all edge membership values. A generalized fuzzy digraph is displayed in Figure 2.

Now, the Vertex centrality v_1 is

$$C_1(v_1) = ((0.1+0.6+0.3)) / (10-1) = 0.11$$

$$C_2(v_1) = ((0.3+0.1+0.4)) / (10-1) = 0.09$$

$$C_3(v_1) = ((0.1+0.2+0.2)) / (10-1) = 0.06$$

Similarly, the vertex centrality of Figure 2 shall be calculated as shown in Table 4

Example:

Finding an active user in the net exam preparation channel of Telegram. Consider the 10 Telegram users in a channel and denoted as $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}$. Also the number of links, messages and videos shared by users in last 3 days is considered as edges.

The following Table represents the edge membership value calculated from corresponding vertices:

Table 6, explains about the Centrality degree of each vertices in Figure 2

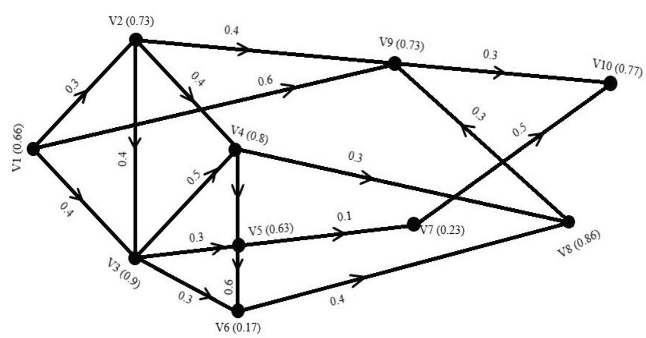


Figure 2. A generalized fuzzy digraph.

Table 4: Vertex membership value (λ) and its Centrality of Figure 2

Vertex	Number of links, messages and videos shared in the past 3days			Member ship value	Centrality
	Links	**Messages**	**Videos**		
v_1	20	17	8	0.66	0.11
v_2	22	11	7	0.73	0.1
v_3	27	19	13	0.9	0.14
v_4	24	16	14	0.8	0.16
v_5	19	7	10	0.63	0.18
v_6	4	2	5	0.17	0.12
v_7	7	1	4	0.23	0.04
v_8	25	20	26	0.86	0.09
v_9	21	14	22	0.73	0.13
v_{10}	13	5	23	0.77	0.06

Table 5. Membership value of edges of Figure 2

Edges	Membership value
(v_1, v_2)	0.3
(v_1, v_3)	0.4
(v_1, v_9)	0.6
(v_2, v_3)	0.4
(v_2, v_4)	0.4
(v_2, v_9)	0.4
(v_3, v_4)	0.5
(v_3, v_5)	0.3
(v_3, v_6)	0.3
(v_4, v_5)	0.7
(v_4, v_8)	0.3
(v_5, v_6)	0.6
(v_5, v_7)	0.1
(v_6, v_8)	0.4
(v_7, v_{10})	0.3
(v_8, v_9)	0.5
(v_9, v_{10})	0.3

Table 6. Centrality degree of all vertices of Figure 2

Vertex	Degree centrality	Degree centrality (N)
v_1	3	0.33
v_2	4	0.44
v_3	5	0.55
v_4	4	0.44
v_5	4	0.44
v_6	3	0.33
v_7	2	0.22
v_8	3	0.33
v_9	4	0.44
v_{10}	2	0.22

6. A Comparative Study on Centrality of GFD and Degree Centrality (N)

In order to compare the degree centrality (N) with the centrality of GFD, a data network had been taken into consideration. The GFD's Centrality is shown in Table 4. The degree centrality for each vertex in Figure 2 is shown in Table 6.

Examining the result.
When compared to GFD centrality, it is seen that degree centrality (N) provides greater values of predictions. Additionally, v_3 has a degree centrality (N) of 0.55 and is the central network node.

Table 7. Comparison of Centrality degree (N) and centrality by GFDof all vertices of Figure 2

Vertex	Centrality by GFD	Degree centrality (N)
v_1	0.11	0.33
v_2	0.1	0.44
v_3	0.14	0.55
v_4	0.16	0.44
v_5	0.18	0.44
v_6	0.12	0.33
v_7	0.04	0.22
v_8	0.09	0.33
v_9	0.13	0.44
v_{10}	0.06	0.22

7. Conclusion

The research on generalized fuzzy digraph unambiguously demonstrated some feasible applications within Telegram. Here, we examined about the node that is active and has the greatest centrality ratings. As a result, the centrality and degree centrality to analyse the active user in a net exam preparation channel in Telegram have been calculated. According to the study, it seems that $\mathbf{v_3}$ is a highly active user in the telegram channel. The declaration that is outlined by the preceding generalized fuzzy digraph indicates the enthusiastic user in the net exam preparation channel in telegram.

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