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Bending Behavior of Clamped Skew Plates

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Abstract

The deflection analyses performed using MSC/NASTRAN on skew plates under different loads such as (concentrated and uniformly distributed loads) for clamped boundary conditions are presented in the present study. The CQUAD4 and CQUAD8 components of MSC/NASTRAN are verified using values from the literature. The CQUAD8 element was employed in the present experiments because it had been found to produce more effective results than the CQUAD4 element. For isotropic skew plates, the variation of deflection with regards to aspect ratio and length to thickness ratio is provided. With an increase in the skew angle, it appears that the deflections decrease.

Keywords: *Bending Analysis, Finite Element, Non-Dimensional Deflection Coefficients, Skew Plates*

1.0 Introduction

The skew plates have an extensive variety of uses in mechanical, civil, shipping, and aerospace engineering applications. Despite the mathematical challenges associated with studying them, they are frequently utilized in contemporary architecture. Skew plates have numerous uses, including in ship hulls, parallelogram slabs in buildings, complex alignment issues in bridge construction, and sweeping wings of aircraft. There are few reliable solutions bending problems of skew plates, and those that do exist are based on approximations.

The dynamic as well as static study of skew plates has received a lot of interest over the past 40 years. Static investigation of skew plates employing either an analytical or numerical solution approach took up a significant portion of this research. The monograph is a good reference on skew plates. He has provided governing equations in oblique coordinates for isotropic skew plates under small deflection theory and presented the solution methods such as trigonometric series, polynomial series and finite difference technique for clamped, simply supported and cantilever skew plates[1]. It is observed that the structural behavior of skew plates is diverse from that of the rectangular plate and the difference widens as the skew angle increases. The central deflections and moments of skew plates are considerably under that of a square, rectangular plate of the same dimensions. As the skew angle increases the convergence problem arises and inaccuracy creeps in.

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Warren W. E. analyzed the clamped and simply supported rhombic plates using point matching scheme[2]. The polar coordinate system was introduced in solution, which adequately displayed Morley's singularity at the obtuse angle[1]. The Boundary Conditions (BCs) have been satisfied in a least square sense. The method gave about 11.5% error compared to that of Morley's results. The clamped skew plates in bending have been studied by Kennedy J. B. assuming the simple polynomial form for deflection function in oblique coordinates[3]. He used the Galerkin's variational approach to evaluate the deflections. For plates of skew angles 15^0, 30^0, 45^0, 60^0, 75^0 and for different aspect ratios new results were presented. The experimental results obtained for two aluminum plates of skew angles 30^0 and 55^0, loaded with water. The results were obtained for different water levels and compared with the theoretical ones. Agarwal[4] described the presentation of conformal mapping method to parallelogram plates. The procedure was validated against that of Morley's[1]. For various skew plates, new results were obtained for concentrated load acting at the center and at an arbitrary point of the plate and for Uniformly Distributed Load (UDL).

By utilizing distinctive parameters for the various modes of vibration of CC (clamped-clamped) beams, Iyenger and Srinivasan have provided a solution to the issue of uniformly loaded clamped skew plates[5]. The outcomes for various skew angles were evaluated in comparison to those from Morley[1]. For highly skewed plates the results obtained by Iyenger and Srinivasan are very high compared to that of Kennedy K. B.[3]. Dawe analyzed rhombic cantilever plates with parallelogram finite element and two types of meshes, 4X4 and 5X5 were used for convergence check[6]. The central deflection of skew plates of 200, 400, 600 are obtained, the 5X5 mesh was graded near the obtuse corner of the clamped plate. This was to improve the accuracy of the numerical values.

Kennedy[7] extended his earlier work, 3-parameter solution by presuming a deflection function in oblique coordinates. Several outcomes from his investigation were compared with that of Iyenger and Srinivasan[5]. Discrepancy in the results was observed, which increases as the skew angle increases and decline as the aspect ratio enhances. The finite element solution was used for four noded rectangular element for skew plates in bending[8].

In the element formulation, bi-cubic interpolation (1^{st} order Hermite) function in terms of oblique coordinates was used. Two types of FE mesh (one uniform and the other graded) were used for all the skew angles in order to moderate the erratic influence of the bending stresses at the obtuse corner of the clamped plate.

Kennedy and Gupta[9] presented a series of clarifications to the issues on bent orthotropic skew structures. In their study various skew angles, boundary and loading conditions were considered. It was observed that skew angle has greater influence on the moments in orthotropic plate structures under uniform loading than those due to concentrated load. Tham *et al.*[10] adopted B-3 spline finite strip technique to study the bending of skew plates through various skew angles, boundary and loading conditions. For bending analysis of rectangular, skew and triangular plates under generalized normal loading. Ganga and Chaudhary[11] developed a fast converging series solution. The procedure is an arrangement of trigonometric and polynomial functions with undecided coefficients. These coefficients are identified using the Galerkin's approach and satisfying BCs. They provided results for clamped and simply supported plates of varying skew angles. Few researchers have investigated an orthotropic thin skew plates using cubic spline integral method[12-14].

Butalia *et al*[13]. obtained results for skew plates in bending using Heterosis finite element. Transformation of stiffness matrices for the elements lying on the skew edges was performed, which enabled the imposition of BCs tangential and normal to the skew edges. The efficiency of this scheme was verified with that of many researchers. Considerable amount of numerical results has been presented for different BCs and loading conditions. This work gives a good overview of the literature on skew plates in bending. Sengupta[14] employed two types of finite elements to analyze isotropic skew plates in bending. He has noted that even for Morley's acute angle with singularity, improved convergence can be obtained for deflections and moments with usual mesh patterns.

Today, the problems on skew plates is frequently utilized by FEM to assess the presentation of a certain newly created finite element. This study examines investigations on the bending behavior of skew plates utilizing the MSC/NASTRAN CQUAD4 and CQUAD8 elements. The elements' accuracy has been established using values from the literature. The present study examines the impacts of skew angle, aspect ratio, and length to thickness ratios on the non-dimensional central deflection coefficient (W_f) of skew plate.

2.0 Convergence and Validation Studies

2.1 Convergence

Figure 1 depicts the geometry of the skew plate using both local and global coordinate systems, where u and v are the

Table 1. Defection at the Centre of a clamped isotropic skew plate with UDL (uniformly distributed load)

Mesh size	Element Type	Skew Angle, α		
		15^o	30^o	45^o
(4X4)	QUAD-8	1.0912	0.7395	0.3510
	QUAD-4	1.2219	0.8382	0.4217
(6X6)	QUAD-8	1.1197	0.7628	0.3703
	QUAD-4	1.2139	0.8446	0.4283
(8X8)	QUAD-8	1.1232	0.7684	0.3754
	QUAD-4	1.1910	0.8243	0.4141
(10X10)	QUAD-8	1.1247	0.7701	0.3771
	QUAD-4	1.1738	0.8095	0.4036
(12X12)	QUAD-8	1.1250	0.7706	0.3778
	QUAD-4	1.1617	0.7996	0.3968
(14X14)	QUAD-8	1.1252	0.7708	0.3780
	QUAD-4	1.1534	0.7930	0.3924
(16X16)	QUAD-8	1.1252	0.7708	0.3782
	QUAD-4	1.1476	0.7882	0.3893
(18X18)	QUAD-8	1.1252	0.7708	0.3782
	QUAD-4	1.1432	0.7849	0.3871
(20X20)	QUAD-8	1.1252	0.7710	0.3782
	QUAD-4	1.1401	0.7824	0.3855
Morley[1]	Analytical	1.121	0.769	0.377
Iyengar *et al.*[5]	FEM	1.123	0.767	0.376
Butalia *et al.*[12]	FEM	1.122	0.768	0.375
Reddy[18]	FEM	1.123	0.769	0.377

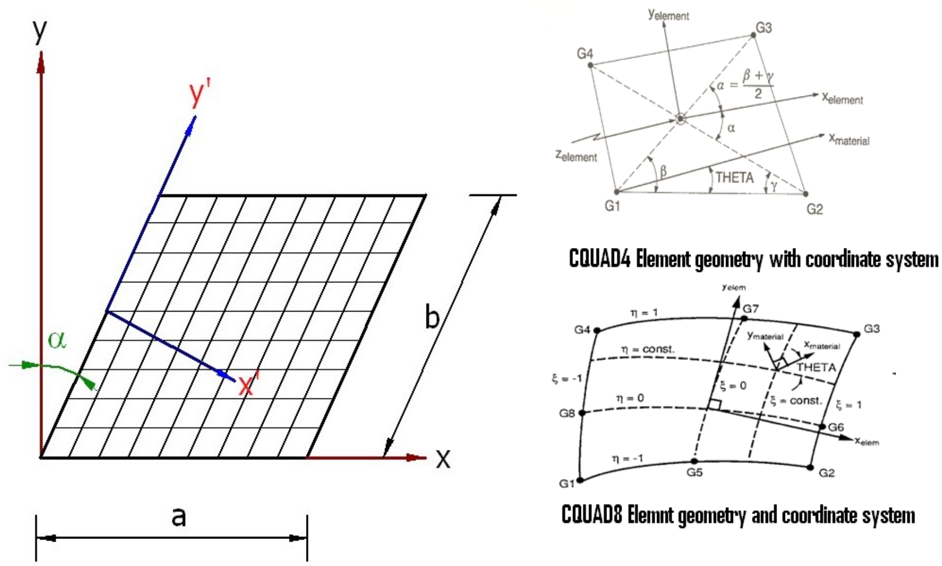

Figure 1. The Skew Plate Using Finite Element Mesh and Elements Coordinate System has both global and local coordinate systems.

x- and y-directional displacement variables, respectively. It is impossible to directly apply the displacement BCs because u and v are angled toward the skew edges. This leads to the selection of a local coordinate system (x, y) that is normal and tangential to the skew edges.

Studying the convergence of the outcomes is crucial to determine the ideal number of elements needed in the finite element model and to obtain accurate and trustworthy results. The convergence study has been performed on clamped(C-C-C-C) skew plates having aspect ratio (=a/b) of 1.0 and skew angles 0°, 15°, 30° and 45° using CQUAD4 (four-noded) and CQUAD8 (eight-noded isoparametric curved shell element) elements of MSC / NASTRAN. Particulars of convergence study are presented in Table 1.

2.2 Validation Check

By comparing the values for the non-dimensional deflection coefficient (W_d) produced in this work with those accessible in the literature, the validation for the finite elements CQUAD4 and CQUAD8 was carried out. Table 2 for isotropic clamped (C-C-C-C) (C2) skew plates shows the same information. Table 2 shows that the results obtained with the CQUAD8 element are more in line with the values found in the literature than the results obtained with the CQUAD4 element.

The clamped (C-C-C-C) skew plate with Uniformly Distributed Load (UDL) and concentrated load were both taken into consideration in this work. The non-dimensional coefficient that represents the transverse displacements (Wf) is defined as follows:

$$\text{Isotropic plate with UDL } = W_f = \frac{wD}{qa^4}$$

$$\text{Isotropic plate with concentrated load } = W_f = \frac{wD}{Qa^2}$$

2.2.1 Clamped Skew Plate under Point Load

An analysis of a clamped skew plate with all edges clamped for different skew angles (α) is performed. Using the current finite element scheme, the non-dimensional central displacements of a clamped plate subject to a concentrated load in the center are evaluated. For isotropic skew plates with skew angles of 0, 15, and 30 degrees investigated thoroughly are the relative thickness and the 45° aspect ratio of 0.5 to 2.5. Table 3 displays the central displacements as non-dimensional coefficients (Wd). Figures 2 to 6 show the deflection profile for various skew degrees along the axis 'x'.

Table 2. Clamped isotropic skew plate with uniformly distributed load

Researchers	Method of Solution	Non dimensional central deflection (Wf) X10^{-3}			
		Skew angle (α)			
		0^{o}	15^{o}	30^{o}	45^{o}
Morley[1]	Analytical Method	-	1.121	0.769	0.377
Iyengar *et al.*[5]	Analytical Method	-	1.123	0.767	0.376
Butalia *et al.*[15]	Mindlin nine-noded Heterosis finite element	-	1.122	0.768	0.375
Kobayashi *et al.*[17]	Analyticalmethod/ trigonometric series	-	1.1229	-	0.37686
Reddy[18]	Finite element method/ high precision triangular element	-	1.123	0.769	0.377
Taylor et. al.[19]	Analytical / classicaldouble cosine series expansion	1.2653	-	-	-
Present	CQUAD8	1.268	1.125	0.770	0.377

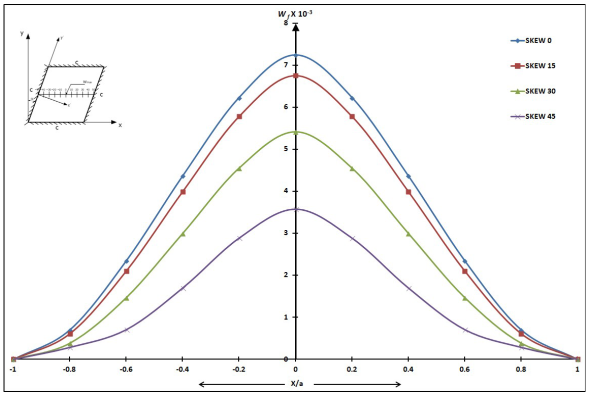

Figure 2. Deflection profile for isotropic skew plates that are clamped and applied to a point load (a/b=0.5, a/t=100).

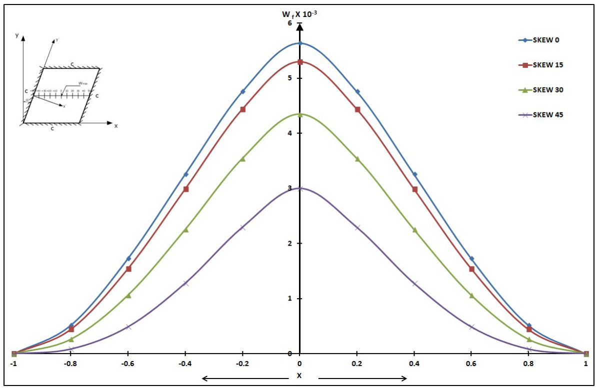

Figure 3. Profile of deflection for clamped isotropic skew plates with point load. (a/b=1.0, a/t=100).

In addition, for isotropic skew plates with skew angles of =0°, 15°, 30°, and 45°, aspect ratios of 0.5 to 2.5 and comparative thickness have undergone extensive examination and are shown in Table 3 as non-dimensional coefficients (W_f). Figures 2 to 3 for clamped under focused load show the deflection profile for different skew angles along the axis ‘x’.

Table 3. Clamped Isotropic Skew Plates with the concentrated load

Aspect Ratio (a/b)	Length-to-Thickness Ratio (a/t)	Non dimensional central deflection (Wf) X10^{-3}			
		Skew angle(α)			
		0^o	15^o	30^o	45^o
0.5	1000	7.219	6.730	5.356	3.548
	500	7.229	6.734	5.388	3.549
	100	7.240	6.753	5.413	3.572
	50	7.311	6.822	5.479	3.635
	20	7.789	7.292	5.921	4.041
1.0	1000	5.604	5.267	4.315	2.967
	500	5.611	5.268	4.316	2.969
	100	5.632	5.293	4.340	2.991
	50	5.708	5.367	4.412	3.058
	20	6.234	5.881	4.903	3.513
1.5	1000	3.129	2.927	2.368	1.585
	500	3.129	2.929	2.369	1. 601
	100	3.155	2.952	2.392	1.612
	50	3.231	3.028	2.465	1.682
	20	3.767	3.555	2.970	1.924
2.0	1000	1.807	1.686	1.345	0.901
	500	1.808	1.689	1.355	0.902
	100	1.833	1.711	1.379	0.924
	50	1.910	1.787	1.452	0.992
	20	2.440	2.316	1.958	1.459
2.5	1000	1.158	1.080	0.867	0.576
	500	1.159	1.082	0.868	0.577
	100	1.184	1.105	0.892	0.592
	50	1.261	1.182	0.965	0.667
	20	1.798	1.711	1.470	0.814

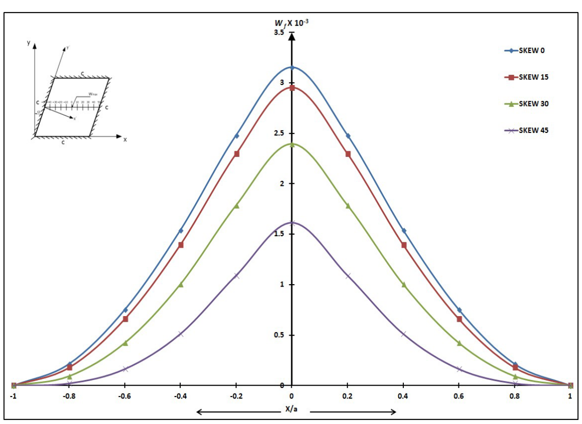

Figure 4. Deflection profile for clamped isotropic skew plates with point load. (a/b=1.5, a/t=100).

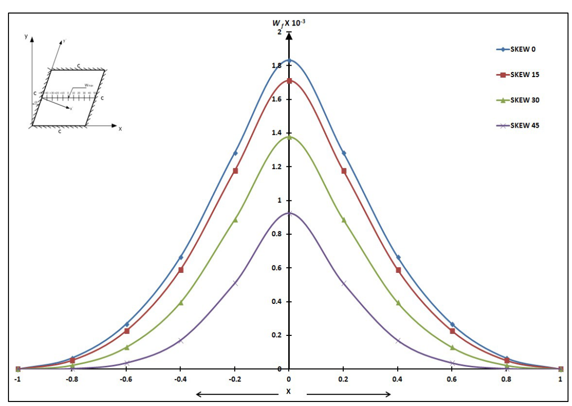

Figure 5. Deflection profile for clamped isotropic skew plates with point load. (a/b=2.0, a/t=100).

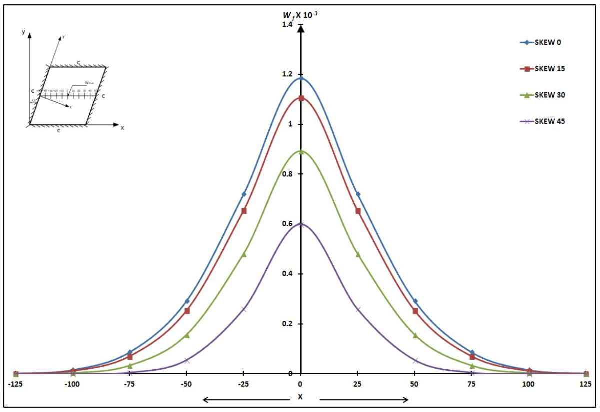

Figure 6. Deflection profile for clamped isotropic skew plates with point load. (a/b=2.5, a/t=100)

2.2.2 Clamped Skew Plate under UDL

The center displacements for isotropic skew plates with skew angles of 0°, 15°, 30°, and 45° are shown in Table 4 as non-dimensional coefficients (Wd). In Table 2, the same values are compared to values from the literature. Table 4 when examined reveals the following. The outcomes of analytical [1, 2, 5, 20,22] and finite element [15, 18] solutions are contrasted with the current values. The values derived utilizing the 8 - noded quadratic plate elements in the current FEM approach are always nearer to analytical values. Figures 7 to 11 show the deflection profile with different skew degrees along the axis 'x'.

Table 4. Clamped isotropic skew plates with uniformly distributed load

Aspect Ratio (a/b)	Length-to-Thickness Ratio (a/t)	Non dimensional central deflection (Wf) X10^{-3}			
		Skew angle(α)			
		0^{0}	15^{0}	30^{0}	45^{0}
0.5	1000	2.532	2.216	1.448	0.652
	500	2.535	2.215	1.449	0.6527
	100	2.539	2.219	1.452	0.6538
	50	2.547	2.230	1.460	0.6593
	20	2.623	2.308	1.517	0.6975
1.0	1000	1.2655	1.122	0.768	0.375
	500	1.265	1.123	0.769	0.376
	100	1.268	1.125	0.770	0.377
	50	1.275	1.133	0.776	0.382
	20	1.327	1.181	0.816	0.410
1.5	1000	0.433	0.3824	0.2554	0.1201
	500	0.4338	0.3825	0.2563	0.1208
	100	0.4354	0.3838	0.2573	0.1221
	50	0.4399	0.384	0.2608	0.1231
	20	0.4706	0.4170	0.2844	0.1392
2.0	1000	0.1580	0.138	0.090	0.04074
	500	0.1583	0.1385	0.091	0.04076
	100	0.1592	0.1592	0.092	0.0412
	50	0.1619	0.1618	0.093	0.04252
	20	0.1808	0.175	0.107	0.05202
2.5	1000	0.06684	0.05824	0.0370	0.01673
	500	0.0668	0.05828	0.0376	0.01676
	100	0.06743	0.05879	0.0380	0.017
	50	0.06918	0.5886	0.039	0.01788
	20	0.08141	0.58872	0.048	0.0239

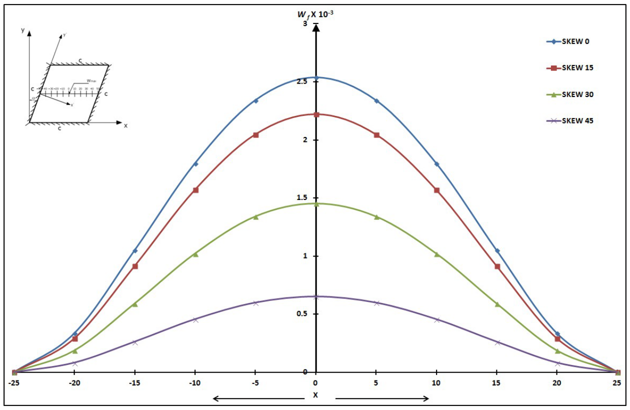

Figure 7. Deflection profile for clamped isotropic skew plates with uniformly distributed load (a/b=0.5, a/t=100).

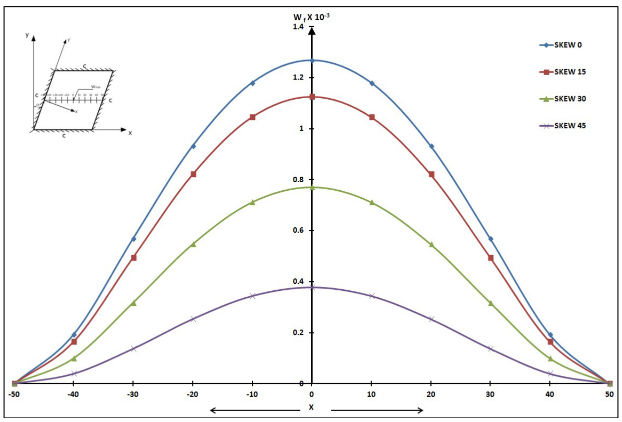

Figure 8. Deflection profile for clamped isotropic skew plates with uniformly distributed load (a/b=1.0, a/t=100).

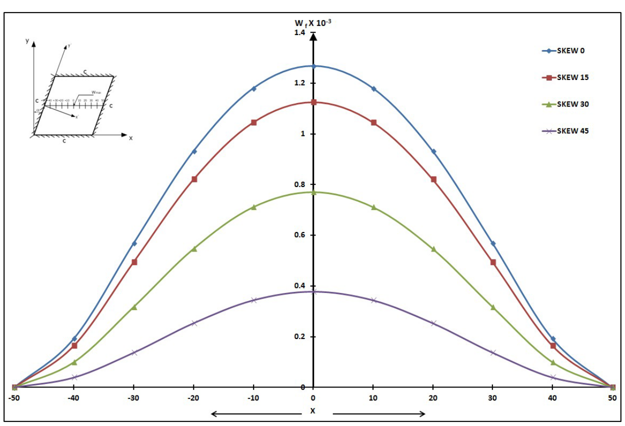

Figure 9. Deflection profile for clamped isotropic skew plates with uniformly distributed load (a/b=1.5, a/t=100).

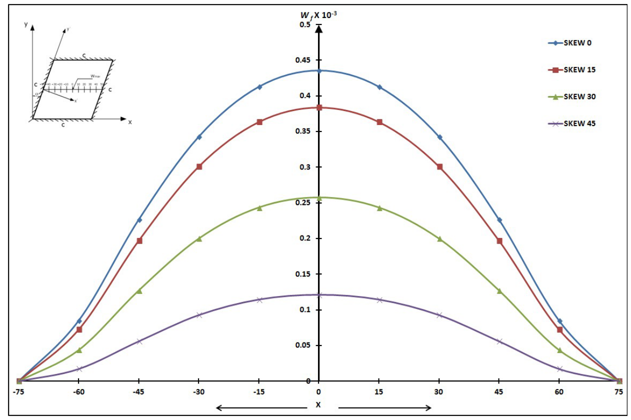

Figure 10. Deflection profile for clamped isotropic skew plates with uniformly distributed load (a/b=2.0, a/t=100).

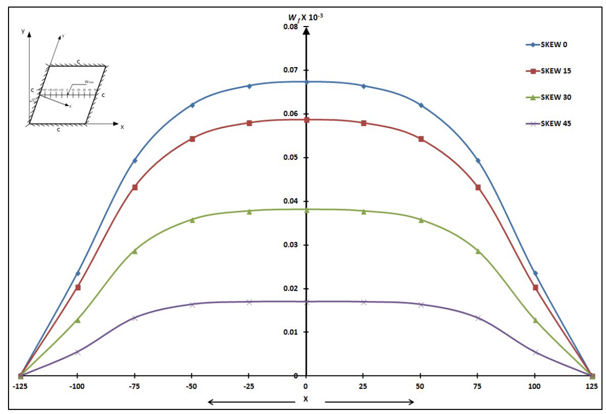

Figure 11. Deflection profile for clamped isotropic skew plates with distributed load (a/b=2.5, a/t=100).

3.0 Conclusions

It has been found that the current eight noded isoparametric finite element model performs superbly for both uniformly distributed and focused loading, even for significant angles of skew. The range of skew angles taken into consideration in this study is up to 45^o. This is because skew plated structures often have skew angles of at least 45 degrees in reality. The numerical findings shown for various skew angles and support circumstances will not only demonstrate the usefulness of the eight-node iso-parametric element now in use, but will also serve as a ready reference for future researchers working in this field.

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Nomenclature:

a	: Plate length
b	: Plate width
t	: Plate thickness
E	: Modulus of elasticity
D	: Plate bending rigidity, $Et^3/12(1-v^2)$
q	: Uniformly distributed load
Q	: Concentrated load
W	: Deflection
W_f	: Non-dimensional deflection coefficient
α	: Skew angle
v	: Poisson's ratio