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Mixed Convection of a Hybrid Nanofluid Flow with Variable Thickness Sheet

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Abstract

The heat transfers of a hybrid nanofluid flow with steady, mixed convection over a variable thickness is investigated in the present work. The non- linear PDE's of the physical model are converted into ODE's by means of suitable similarity transformations. The subsequent ODE's are solved using MATLAB and shown the effects graphically for the parameters like wall thickness, mixed convection, velocity index for both nf and hnf, Nusselt number and Skin friction for hybrid nf.

Keywords: *Hybrid Nanofluid, Mixed Convection, Similarity Transformation, Variable Thickness.*

1.0 Introduction

Nano fluids are a class of fluids with which the rate of heat transfer process plays a vital role, owing to this it has acknowledged in several industrial and engineering usage such as cooling of engine oils, devices of exchanging heat, thermal power plants etc.

The enhancement of thermal conductivity was initially studied by Choi[1], preparing a Nanofluid by means of Nanoparticles. In view of the applications related to diverse fields like pharmaceutical, environmental and medicinal, Nano fluids are contemplated with size, shape and concentration. Based on the rheological properties of the Nano elements the thermal conductivity of the nanofluid was deliberate by Chen[2] *et al.* Kwak[3] *et al.* piloted a study to know the enhancement of heat transfer and thermal conductivity of a Nano fluid.

Several of the authors have intended their study on Nano fluids with regards to heat transfer and thermal conductivities. Recently some of the researchers are conducting many experiments by suspending two types of nano particles in the base fluid termed as "Hybrid Nanofluid" or Composite Nano fluids. With the appropriate combination of different nano particle suspension, some of the constructive features can be enhanced because of their synergistic effect. Some of the applications of hybrid nanofluid are in the grounds of medical, naval structures etc. In view of this Syam sunder[4] *et al.* intended his study in preparing the hybrid Nanofluid with different base fluids, thermal properties and heat transfer characteristics. Hayat[5] *et al.* has dedicated his study towards hybrid Nanofluid by considering a three dimensional, rotating fluid through a stretching surface by combining silver, copper oxide and water. Nurul[6] *et al.* has done a numerical work on magneto hydrodynamic, steady 2D, mixed convection immobility point flow of a hybrid nanofluid past a perpendicular flat plate. Uma devi[7] *et al.* premeditated numerically the impact of heat transfer of nanofluid prepared with copper, aluminum oxide and water through a stretching sheet.

Surdarshan Reddy[8] *et al.* addressed flow inside a square cavity a entropy generation and heat transfer by the solicitation of magnetic field. Momin[9] conducted experiments on mixed convection laminar flow of

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a hybrid Nanofluid through an inclined tube and investigated some of the experimental results. Suresh[10] *et al.* manufactured water based hybrid Nanofluid and studied its thermo physical properties by means of two step method. Chamkha[11] *et al.* has done a numerical work on hybrid Nanofluid with water as a base and analyzed the conjugate natural convection effects in a semicircular cavity. Izadi[12] *et al.* studied the free convection of a hybrid Nanofluid by considering an inversed T- shaped enclosure in a partitioned porous media. Further the authors (13) to (20) dedicated their study towards hybrid Nanofluid by considering different effects. Botha *et al.* [21] did an extensive work on oil based Nanofluid with hybrid structures of silver Nano particles in the presence of silica.

The idea of boundary layer flow through a surface having variable thickness was first introduced by Lee[22]. Subsequently Fang[23] *et al.* analyzed analytically and numerically the boundary layer flow in a stretching sheet. Subhashini[24] *et al.* obtained the dual solutions through a stretching surface by taking variable thickness. Further Ramesh[25] *et al.* has done an extensive work on variable thickness for a Casson fluid through the stretching surface. Nancy Samuel[26] investigated the triple diffusive stretching of variable thickness sheet in a porous medium.

Recently many authors studied mixed convection hybrid nanofluid with multiple slip, novel shaped porus medium with heat sources and sink, thermal radiation

The novelty of the present study is to investigate numerically the two dimensional steady, mixed convection flow of a hybrid Nanofluid over a variable thickness sheet. To solve the governing equations similarity transformation is used and the subsequent equations are solved using the MATLAB package bvp4C. The particular hybrid nano fluids that we considered in our paper are aluminum oxide, cupric oxide, silver oxide and magnesium oxide. The key motivation for considering these metallic oxides as nanoparticle's are

- Aluminum oxides hardness used in lab equipment and tools like crucibles, furnaces. Due to its high melting and boiling points, it is used in manufacture of electric furnace, Alumina films, spark plug insulators, micro-electric substrates and insulating heatsinks.
- Cupric oxide is used in the welding process, manufacture of lithium batteries.
- Silver oxide is used in batteries, aerospace and fuel cells.
- Magnesia crucibles have found application in the super alloy industry, nuclear industry etc.

2.0 Mathematical Formulation

Steady two dimensional mixed convection boundary layer flow of a viscose incompressible fluid over a semi-infinite vertical stretching sheet with variable thickness is considered. The x-axis is chosen in the direction of the sheet motion and y-axis is normal to it. It is assumed that the impermeable sheet stretches with velocity with $U_w (x)=U_0 (x+b)^m$, where U_0 is a constant, , b is the stretching sheet related constant and m is the velocity power index . for the given profile the variable thickness of the sheet is given by $y=a\ (x+b)^{(1-m))/2}$, where a is the co-efficient related to the stretching sheet. The speeding up or slowing down of the sheet causes escalation or diminution in the width of the sheet with space from the slot. The change in the thickness of the sheet is reliant on the value of the velocity power index m. The value of constant a is taken small so that the sheet can be sufficiently tinny as shown in Figure a.

The governing boundary layer equations of the stream is given by (6)

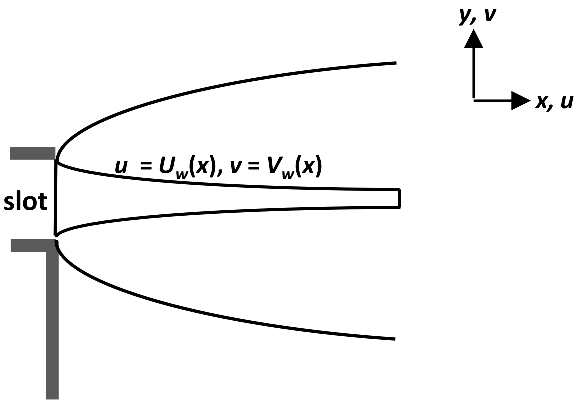

Figure a. Flow geometry.

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 y}{\partial x^2}+\frac{(\rho\beta)_{hnf}}{\rho_{hnf}}(T-T_\infty)g \tag{2}$$

Boundary conditions (26)

$$u = u_w(x),$$

v=0, T=T$_w$ (x) as

$$y = a\,(x+b)^{(1-m)/2}$$

u→0, T→T$_\infty$, as y→y$_\infty$ (4)

In order to obtain similarity solutions, the velocity, temperature at the wall are considered in the following form

$$u_w(x) = U_0(x+b)^m$$

$$T_w(x) = T_\infty + T_0(x+b)^{2m-1} \qquad (5)$$

The following similarity transformations [26]

$$\eta = y\left(\frac{U_0}{\upsilon_f}\right)^{1/2}(x+b)^{(m-1)/2} \qquad (6)$$

$$\psi = \left(\upsilon_f U_0\right)^{1/2}(x+b)^{(m+1)/2} f(\eta) \qquad (7)$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \qquad (8)$$

are used in Equations (1)-(3), so that velocity components and are obtained as follows

$$u = U_0(x+b)^m f'(\eta) \qquad (9)$$

$$v = -\left(\upsilon_f U_0\right)^{1/2}(x+b)^{(m-1)/2}\left[\frac{(m+1)}{2}f(\eta) + \frac{(m-1)}{2}\eta\, f'(\eta)\right]$$

(10)

On replacing the similarity variables (6-8), Equations (2) & (3) are abridged to the following set of ODE's

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + \left(\frac{m+1}{2}\right) f f'' - m f'^2 + \frac{\beta_{hnf}}{\beta_f}\lambda\,\theta = 0$$

(11)

$$\frac{1}{P_r}\,\frac{k_{hnf}/k_f}{(\rho\, C_p)_{hnf}/(\rho\, C_p)_f}\theta'' + \left(\frac{m+1}{2}\right) f\,\theta' - (2m-1) f'\theta = 0$$

(12)

subject to the transformed boundary conditions

$$f(\chi) = \left(\frac{1-m}{1+m}\right)\chi, f'(\chi) = 1, \theta(\chi) = 1 \qquad (13)$$

f' (η)=0, θ(η)=0 asη→η$_\infty$ (14)

Equations (11) & (12) with boundary conditions (13) & (14) are non-linear differential equations with the field [X,∞]. In order to simplify the computation and to convert the field into [0,∞], describe f(ξ)=f(η-χ)=h(η), θ(ξ)=θ(η-χ)=θ(η), The similarity equations become

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} h''' + \left(\frac{m+1}{2}\right) h h'' - m h'^2 + \frac{\beta_{hnf}}{\beta_f}\lambda\,\theta = 0$$

(15)

$$\frac{1}{P_r}\,\frac{k_{hnf}/k_f}{(\rho\, C_p)_{hnf}/(\rho\, C_p)_f}\Theta'' + \left(\frac{m+1}{2}\right) h\,\Theta' - (2m-1) h'\Theta = 0$$

(16)

With the boundary conditions

$$h(\xi) = \chi\left(\frac{1-m}{1+m}\right) \quad ,$$

h' (ξ)=1, θ(ξ)=1 at ξ=0 (17)
h' (ξ)=0, θ(ξ)=0 as ξ→∞ (18)

The local skin friction coefficient C$_f$ and the local Nusselt number Nu$_x$ which are defined as

$$C_f = \frac{\tau_w}{\rho_f\, u_w{}^2}, Nu_x = \frac{q_{w(x+b)}}{k_f(T_w - T_\infty)}$$

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial x}\right)_{y=0}, q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (19)$$

(20)

$$Re_x{}^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} h''(0)$$

$$Re_x^{-1/2}\ Nu_x = \frac{-k_{hnf}}{k_f}\ \theta''(0) \qquad (21)$$

3.0 Results and Discussion

Equations (15) and (16) were solved numerically by the using MATLAB. Figure 1 & Figure 2 illustrates the influence of wall thickness parameter χon velocity and temperature profile for different nf and Hnf for values of pr=6.2, m = 0.1 χ = 0.1. Figure 1a & Figure 2a shows the stimulus of wall thickness parameter on velocity and temperature for the nanofluid Al_2O_3/H_2o and the hybrid nanofluid $Cu\text{-}Al_2O_3/H_2O$ and Figure 1b and Figure 2b displays the impact of wall thickness parameter on velocity and temperature for the nanofluid MgO/H_2O and the hybrid nanofluid $Ag\text{-}MgO/H_2O$. It can be noticed that in Figure 1a & Figure 1b increase in wall thickness parameter of the stretching sheet acts as an element which impedes the flow and in Figure 2a and Figure 2b thermal BLT decreases this happens due to the point that movement of heat is less which forms profuse regions to the flow than the stripper regions. In comparison with Figure 2a and Figure 2b it is observed that thermal BLT of Hnf is more than nf it is around 3 and 2.7 respectively. Reason behind this is thermal conductivity of Hnf is more related to the nanofluid. Figure 3 & Figure 4 shows the effect of velocity index parameter m = on velocity and temperature profile for different nf and hybrid Hnf for fixed values of pr= 6.2, m = 0.1 χ = 0.1. It is perceived that as the values of velocity index parameter m increases from m = 1 to m = 4 the velocity and thermal BLT decreases in case of both nf and Hnf respectively. In Figure 3a and Figure 3b BLT increases from 6 to 5 and in Figure 4a and Figure 4b decreases from 2.5 to 1.7 respectively. Figure 5 & Figure 6 shows the upshot of mixed convective parameter on velocity and temperature profile for two different nf and Hnf for fixed values of pr = 6.2, m = 0.1 χ = 0.1. It is noticed in Figure 6 that increase in the values of mixed convective parameter decreases the thermal BLT in both the cases of nanofluid and hybrid nanofluid respectively. It is observed in Figure 5 that increase in the value of λ, the momentum BLT increases near the surface,

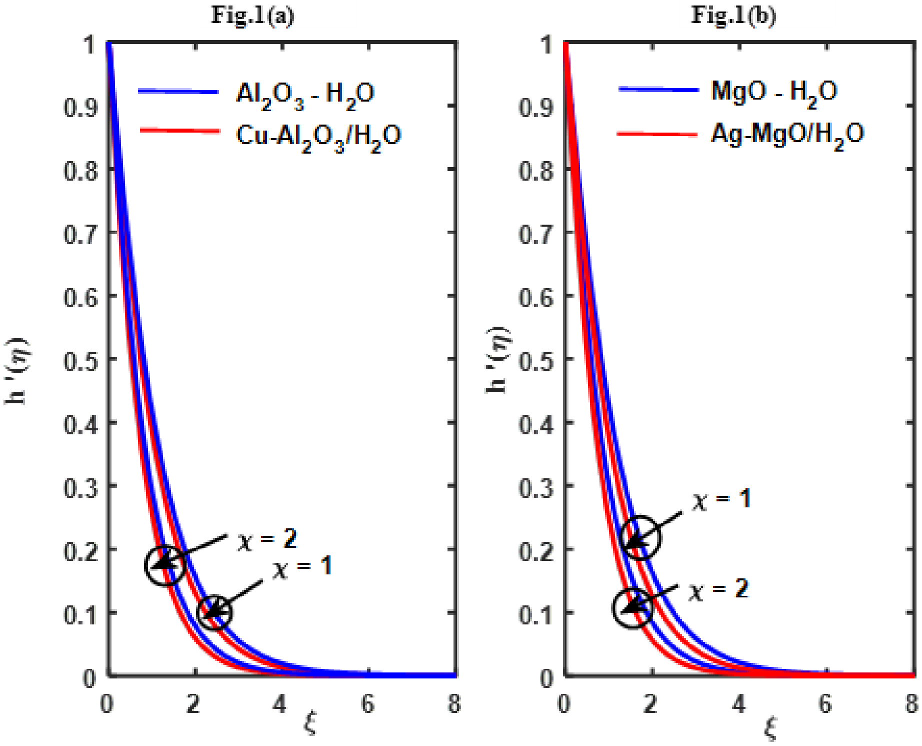

Figure 1. Influence of χ on velocity.

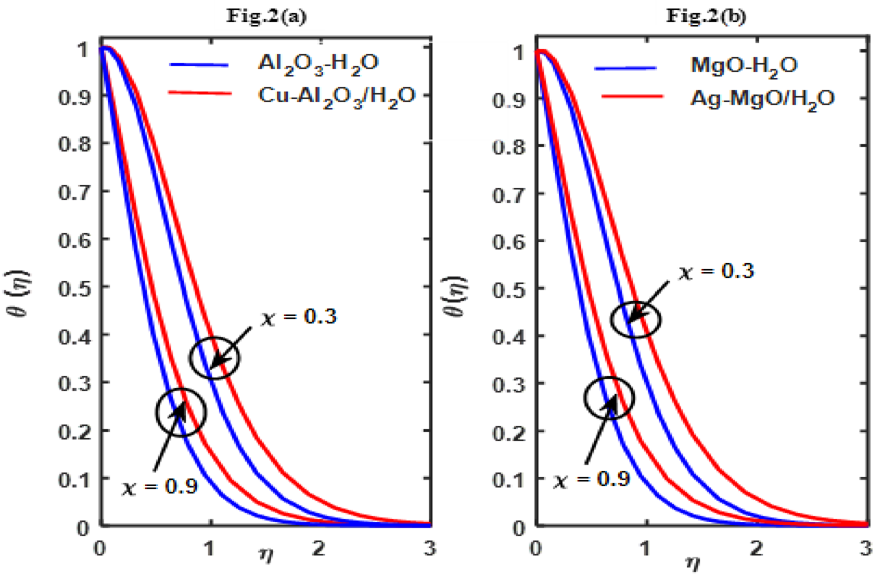

Figure 2. Influence of χ on temperature.

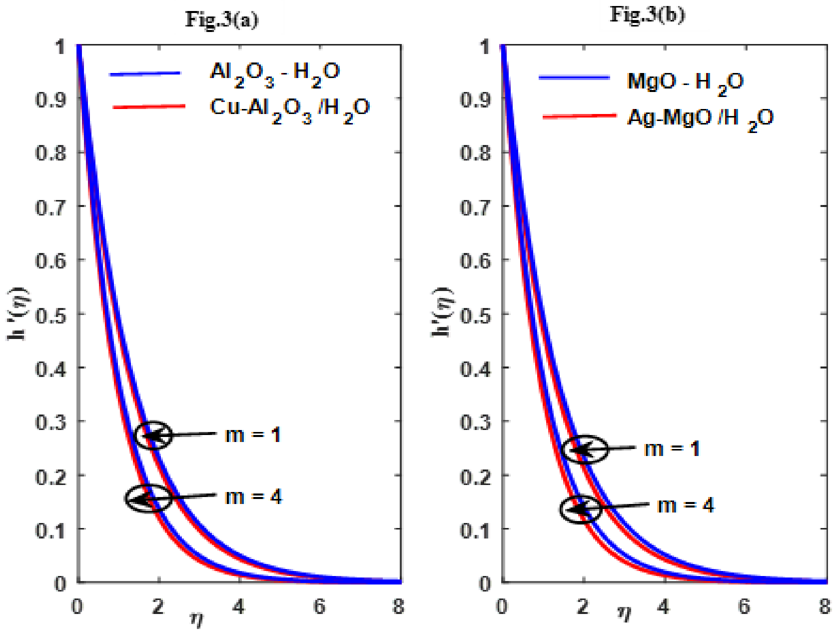

Figure 3. Influence of m on temperature.

this happens due to the fact that high velocity towards neighboring surface conveys more heat out of the surface ,thus drops the thermal BLT. Also in Figure 5 we noticed that velocity overshoot close the plate physically λ>0 means warming of the fluid or conserving of the surface of the plate (assisting flow).An rise in the values of λcan lead to rise in the stimulus of temperature distribution in the velocity stream which sources an improvement of the velocity due to improved convection currents, thus an increase in the BLT. Figure 7 depicts the Variation of

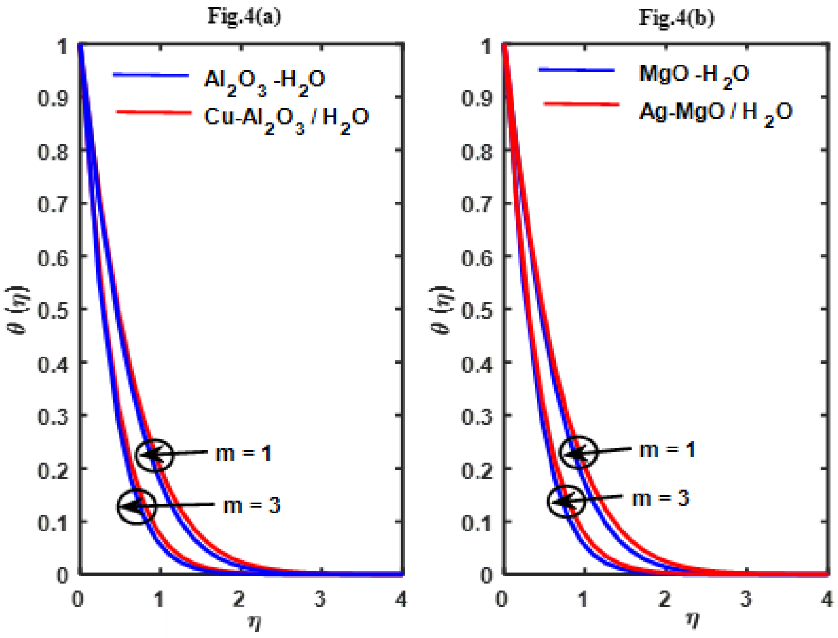

Figure 4. Effect of m on temperature.

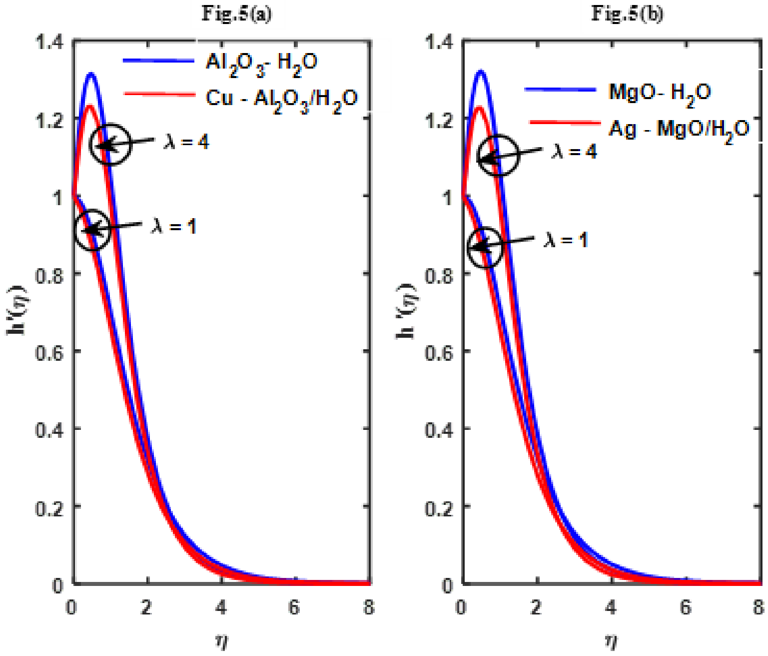

Figure 5. Influence of λ on velocity.

$\theta'(0)$ with λ for dissimilar values of ϕ_2, it is observed that increase in $\phi_2 = 0.02$ to $\phi_2 = 0.05$, heat transfer decreases as mixed convective parameter increase from 1 to 4 this happens due to the thermal BLT rises in case of both hybrid nanofluid $Cu\text{-}Al_2O_3/H_2O$ and $Ag\text{-}MgO/H_2O$ and respectively. Figure 8 shows the deviation of $h''(0)$ with

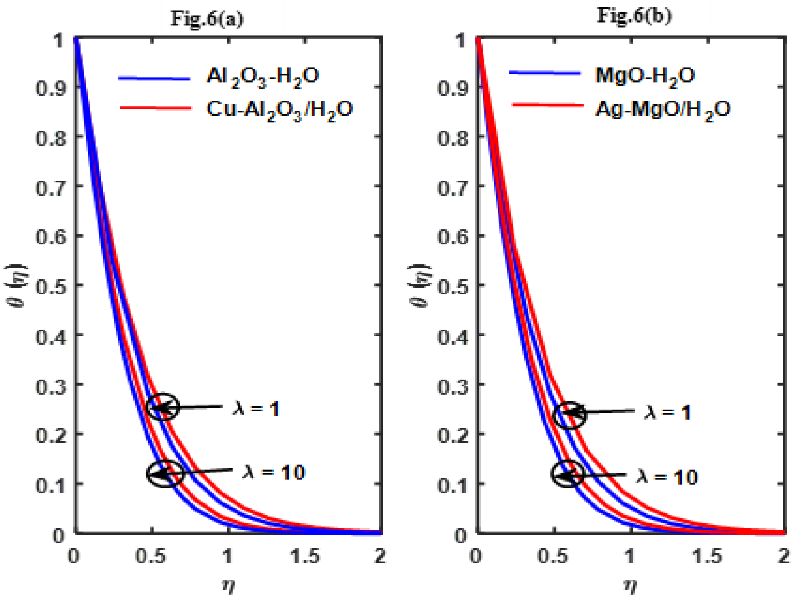

Figure 6. Influence of λ on temperature.

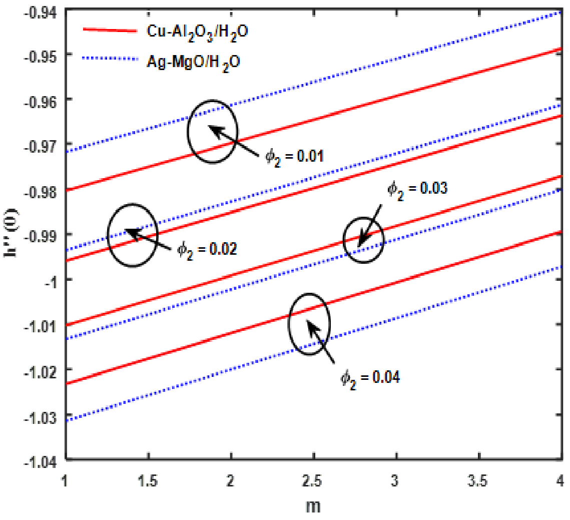

Figure 7. Variation of $\theta'(0)$ with λ for different values of ϕ_2

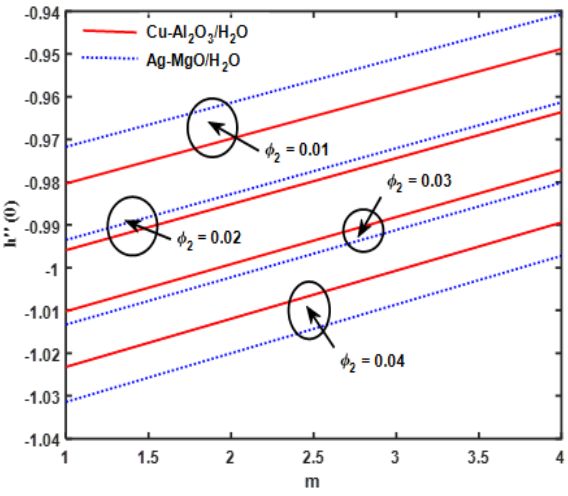

Figure 8. Variation of $h''(0)$ with for different values of ϕ_2

m for diverse values of ϕ_2, as ϕ_2 increasesfrom 0.01 to 0.04 the local skin friction coefficient decreases as velocity power index parameter increase from 1 to 4 in case of both hybrid nanofluid *Cu-Al$_2$O$_3$/H$_2$O* and *Ag-MgO/H$_2$O* respectively.

4.0 Conclusion

The velocity and thermal BLT of various parameters like wall thickness, mixed convection velocity power index parameter have been investigated graphically using BVP4C.The results obtained are as follows:

- Increase in wall thickness parameter decreases the velocity BLT.
- Increase in mixed convection parameter increases velocity and thermal BLT.
- Increase in velocity power index decreases the velocity and thermal BLT.
- Local Skin –friction coefficient decreases as ϕ_2 increases from 0.01 to 0.04.

5.0 Acknowledgment

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Nomenclature

u -velocity components along x	C_p -specific heat at constant pressure
v -velocity components along y	μ_{hnf}-dynamic viscosity of hybrid nanofluid
g - acceleration due to gravity	k_{hnf} - thermal conductivity of hybrid nanofluid
υ -kinematic viscosity	T-Temperature of the fluid
β-volumetric coefficient of thermal expansion	T_∞- Temperature of the ambient fluid
ρ- density of the base fluid	ODE's-ordinary differential equations
ρ_{hnf} -density of the hybrid nanofluid	PDE's-partial differential equations
η_∞- edge of the boundary layer	nf –nanofluid
$\chi = a(U_0/\upsilon)^{1/2}$ - wall thickness parameter	Hnf-hybrid nanofluid
λ- mixed convection parameter	BLT-boundary layer thickness
ϕ_1-nanoparticle volume fractions for Al2O3/MgO (alumina) /(Magnesium oxide)	ϕ_2 nanoparticle volume fractions for Cu (copper)/ Ag(silver)