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Numerical Study of Unsteady Boundary Layer Flow through A Vertical Surface with Convective Boundary Conditions

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Abstract

In the present-day paper the effort is to see the properties of heat allocation over a vertical flat surface for an unsteady mixed, boundary layer convective flow. The non-linear partial differential equations of the present model are transformed in to ordinary differential equations by means of suitable similar transformations. The consequent ordinary differential equations are resolved mathematically by means of MATLAB program bvp4c and discussed the effects of skin friction, suction and injection for temperature and velocity.

Keywords: *Mixed Convection, Similarity Transformation, Suction/Injection, Unsteady Flow*

1.0 Introduction

The improvement of modern technology has stimulated many researchers to work on fluid flow accompanying many other physical phenomena. The study is contributed toward the mixed convection where both forced and free convection will act upon together, and due to this buoyant forces play an important role in the forced flow. The process of mixed convection above a flat surface has established prominence due to its applications in the field of engineering such as in built-up of some of the equipment's like nuclear reactors, heat exchangers etc. In view of this much of the research work is in progress. In literature the work related to steady mixed convection buoyant effects above a flat surface has studied[1].

The Convective heat transfer in fluids has gained importance in the study of geothermal energy recovery and some of the applications are in the recovery of oil in the field, in processing of food, in designing packed bed reactors, the dispersion of chemical contaminants in various process in chemical industry and in the environment. Also the study of convective transport process in porous media is helpful in the deep geological repositories for the disposal of high level nuclear waste.

Owing to the applications of geophysics and heat storage[2] the author has deliberated a numerical study to know the effects of steady boundary layer flow for an impermeable steep surface nearby stagnation point. Due to the fact that the conventional fluids like water, oil, ethylene etc. are not good at heat transmission properties, the insertion of Nano sized particles to the fluid came into existence and which in turn enhanced the heat transfer qualities. In this regard the authors[3-6] contributed their study in the direction of boundary layer flow of mixed convection steady flow of Nano fluids. In later studies different types of Nano sized particles were included in the base fluid and many researchers[7-10] intended to study on hybrid Nano fluids for a mixed convection. The author[11] discussed mixed convection with permeable vertical surface.

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Owing to the applications of Electronics cooling, furnace Engineering, manufacturing solar collectors heat transfer with moving boundaries has deliberated attention in present years due to increasing application in the industries. Many researchers are discussed unsteady fluid flow, Authors[12-15] discussed movement of fluid with mixed convection from rotating vertical cone, surface by the application of magnetic field with different fluids.

Recently discussed double diffusive boundary layer flow for an unsteady mixed convection within incompletely heated enclosure[16]. In the literature it is observed that several of the work is done with respect to steady and unsteady mixed convection boundary flow with Nano and without Nano fluids. The distinctiveness of the present analysis is to ascertain the effects of mixed convection unsteady boundary layer flow of fluid over a vertical flat surface by means of convective boundary.

2.0 Mathematical Formulation

For the flow model, unsteady two dimensional mixed convection boundary layer flow towards the flat surface has been considered. The temperature of the viscous fluid is taken as T_∞. Due to the persistent nature of the fluid having variations in density, the buoyant forces become significant. Throughout the model it is presumed that the movement of the free stream is towards upper part of the solid surface having velocity U_e. The Boussinesq estimate is appealed for fluid possessions to explain density fluctuations with temperature variations, and to combine the temperature field to the flow field. With these conditions leading equations of the model is given by[11]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\,\beta\left(T-T_\infty\right) \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\,\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

The relevant boundary settings for velocity and temperature [11] are

$u=0,\ v=V_w,\ -kT_y=h_f\left(T_f-T_w\right)$ when y takes zero

(4)

$u\to U_e(x,t),\ V\to 0,\ T\to T_\infty$ when y takes ∞.

$V_w < 0$ represents injection velocity & $V_w > 0$ represents suction velocity.

The similarity transformations[12]

$$\eta=\sqrt{\frac{U_e}{x\upsilon}}\,y=y\sqrt{\frac{ax/(1-ct)}{x\upsilon}},\ \ \psi(x,y)=(\upsilon xU_e)^{1/2}x\,f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty} \tag{5}$$

Substituting (5) for Eqs. (1) – (3), Eq. (1) gives the velocity components u and v as

$$u=U_e\,x\,f'(\eta),\ v=-\sqrt{\frac{\nu a}{1-t}}\,[f(\eta)], \tag{6}$$

where η is the similarity variable, prime notation signifies derivatives with respect to η, Ψ is the stream function, θ is the dimensionless temperature .

On substituting new variables (5), Equations (2) & (3) takes the form

$$f'''-f'^2+f\,f''+\lambda(x)\theta-A\left[f'+\frac{\eta}{2}f''\right]=0, \tag{7}$$

$$\theta''+\Pr\left(f-\frac{\eta}{2}A\right)\theta'=0, \tag{8}$$

The boundary conditions transformed using similarity transformations are given by

$f(\eta)=f_w,\ f'(\eta)=1,\ \ \theta(\eta)=m(1-\theta'(0)$ at

$\eta=0$ (9)

$f'(\eta_\infty)=0,\ \ \theta(\eta_\infty)=0$ at $\eta=\eta_\infty,$

$\mathrm{Re}_x=\dfrac{U_e x}{\nu}$ is being local Reynolds number (10)

3.0 Results and Discussion

Equations (7) and (8) were solved numerically by using MATLAB. Figure 1 & Figure 2 depicts the consequence of the suction and injection on velocity and temperature profile respectively. It is observed that in Figure 1 in case of injection the fluid is conceded far from the surface and initiating decline in velocity gradient as it stabs to uphold the similar velocity above the minor region close to the surface and this result is totally upturned in the instance of suction. In Figure 2 it is detected that reductions in the thermal boundary layer wideness with suction increases with injection. Figure 3 and Figure 4 shows the consequence of buoyancy factor on velocity and temperature profile respectively. It is observed that in Figure 3 for $\lambda > 0$ (assisting flow) rise in the values of λ rises the velocity boundary layer thickness with less overshoot because of strong buoyancy effects due to low viscosity of the fluid and in Figure 4 decreases the thermal boundary layer thickness due to low thermal conductivity. Figure 5 presented the effect of convective parameter on temperature profile it is seen that plate temperature of the surface accelerates as the constraint m rises, as $m \to \infty$ result slants to the typical solution for the persistent surface temperature . The outcomes can also be realized from the boundary conditions $\theta(0)=1$ as $m \to \infty$. Figure 6 indicates the impact of Prandtl number on temperature outline it is seen that as the Pr increases from 0.1 to 0.3 the thermal boundary layer width falls from $\eta \approx 12.5$ to $\eta \approx 8$ this happens due to the fact that in the lower values of Pr having high thermal conductivity thereby reduces the thermal boundary layer width. In Figure 7 it is witnessed that growth in unsteady factor drops the thermal boundary layer width which upshots in fall of heat transfer. It is observed that in Figure 8 that as the buoyancy effect increases from $\lambda = 0.1$ to $\lambda = 0.2$ the skin friction decreases when m varies from $m = -0.1$ to $m = 0.1$ this is because of the fact that the velocity boundary layer width improves with assisting flow there by decreases the $f''(0)$ and skin friction co-efficient rises with rise in unsteady parameter.

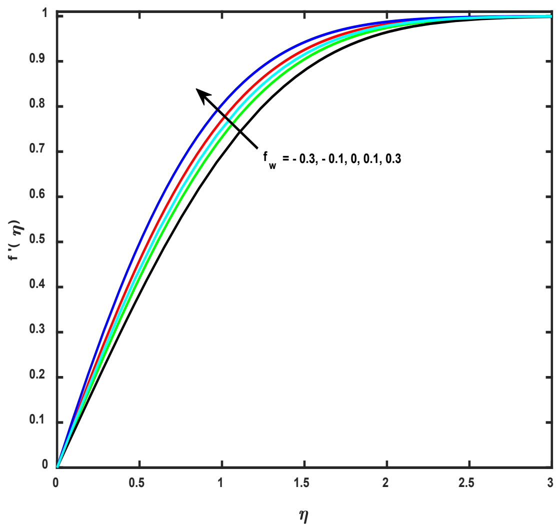

Figure 1. Impact of suction/injection f_w on the velocity profile for the different values of Pr = 7, A = -1, m = 1, $\lambda = 0.2$.

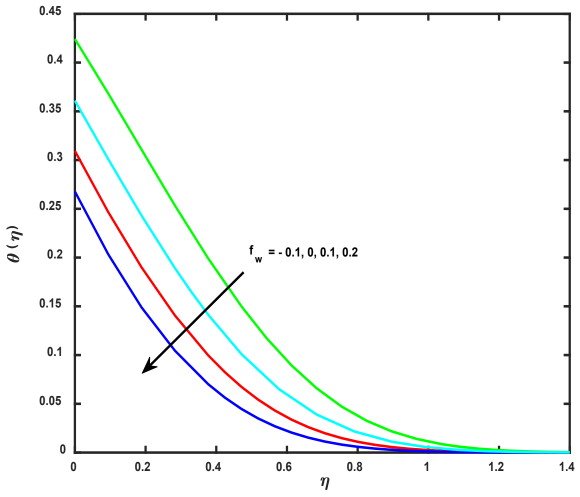

Figure 2. Impact of suction/injection f_w on the temperature profile for the different values of Pr = 7, A = -1, m = 1, $\lambda = 0.2$.

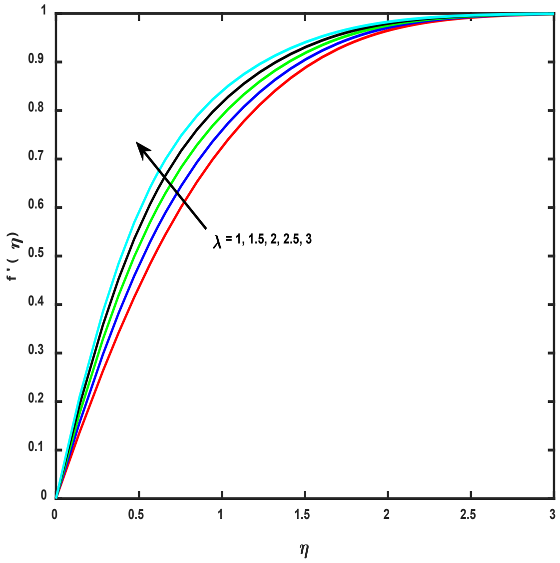

Figure 3. Impact of λ on the velocity profile for the different values of Pr = 7, A = -1, m = 1, $f_w = 0.5$.

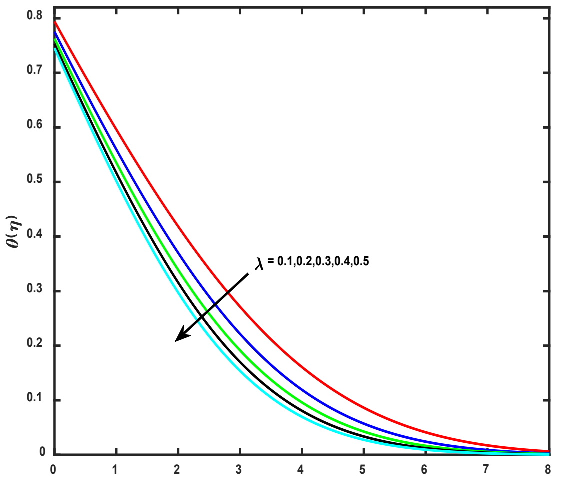

Figure 4. Impact of λ on the temperature profile for the varied values of Pr = 7, A = -0.01, f_w = 0.1, m = 1.

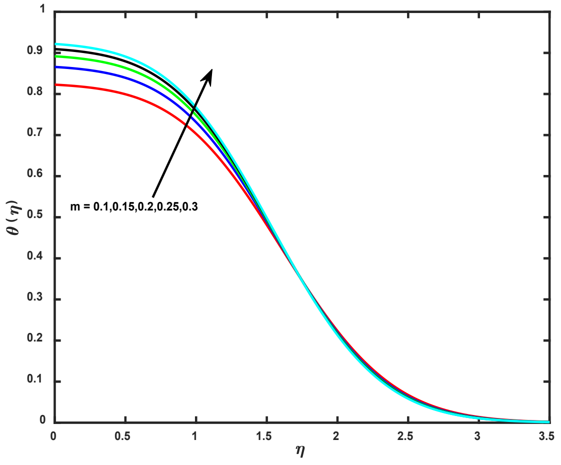

Figure 5. Impact of convective parameter m on the temperature profile for the various values of Pr = 7, A = 0.01, f_w = 0.5, λ = 0.2.

4.0 Conclusion

- Thermal boundary layer width drops with the enhancement of Prandtl number and unsteady parameter.
- Skin friction co-efficient decreases with growth in unsteady parameter.

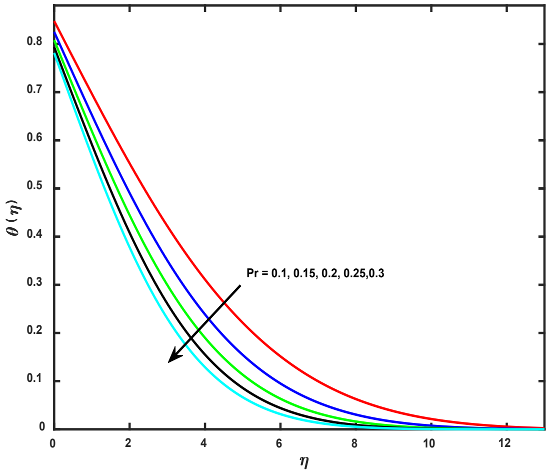

Figure 6. Impact of Prandtl number Pr on the temperature profile for the different values of A = 0.01, f_w = 0.1, m = 1, λ = 1.

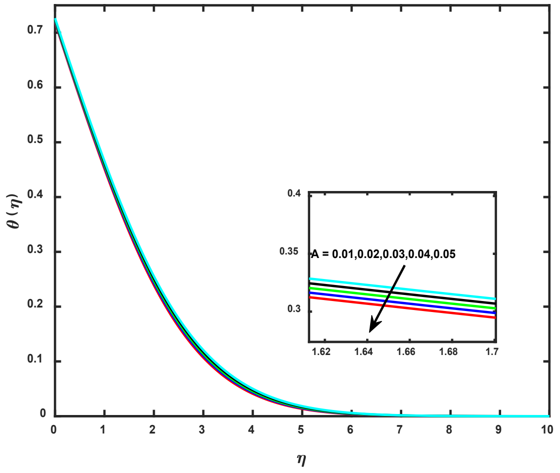

Figure 7. Impact of Unsteady parameter A on the temperature profile for the different values of Pr = 0.7, f_w = 0.1, m = 1, λ = 1.

- Increase in convective parameter increase the heat transfer near the wall
- Velocity boundary layer thickness decreases with injection increases with suction.

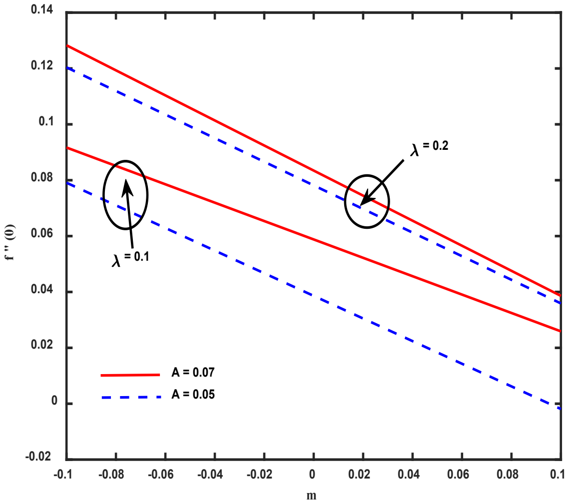

Figure 8. Deviation of skin friction coefficient $f''(0)$ on m for the various cases of Pr = 7, A = -0.01, f_w = 0.5, λ = 0.2.

5.0 Acknowledgment

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6.0 References

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Nomenclature

u- component of velocity in x

v- component of velocity in y

υ - kinematic viscosity

g - acceleration with respect to gravity

β- Thermal expansion co-efficient

α- Thermal conductivity

$f_w = 0$ suction/injection parameter

λ-dimensionless mixed convection parameter

Pr- Prandtl number

m - convective parameter.