Print ISSN : 0022-2755
Journal of Mines, Metals and Fuels
Contents available at: www.informaticsjournals.com/index.php/jmmf

Non-Darcy-Benard Double Diffusive Marangoni Convection with Dufour Effect in a Two-Layer System

R. Sumithra[1], Shivaraja. J. M.[2*], T. Arul Selvamary[3]

[1]Associate Professor, Head of the Department,
[2&3]Research Scholar, Department of UG, PG Studies & Research in Mathematics, Government Science College (Autonomous), Nrupathunga University, Nrupathunga Road, Bengaluru 560 001, Karnataka.
**Corresponding author: shivarajajm95@gmail.com*

Abstract

Two - component / Double Diffusive Marangoni (DDM) convection with Dufour effects, in a two-layered system, has been studied analytically by using Darcy – Brinkmann Model. For fluid layer, the upper boundary is free with surface tension depending on both temperature and concentration, for porous layer the lower boundary is rigid and both the boundaries are insulating to both heat and mass. At the interface, the normal velocity, normal stress, shear stress, mass, mass flux, heat, heat flux is assumed to be continuous. The effect of different physical parameters on DDM convection is investigated in detail and results are presented graphically. The effect of Dufour parameter, which plays vital role in diffusion-thermal process, when the energy flux due to mass gradient appears, on DDM convection in a two- layer system, has been explored.

Key words: *Dufour effect, Double Diffusive, Marangoni Convection.*

1.0 Introduction

Convection in fluids, driven by two different density gradients, which have different rates of diffusion is called Double Diffusive convection. Marangoni convection caused by means of the surface tension gradient because of the gradients of both temperature and concentration while heat and mass transfer happens concurrently in a transferring fluid.

The relation among the fluxes and using potentials are of extra elaborate in nature, it been observed that an energy flux caused by a concentration or salinity gradient is called the diffusion-thermal method or Dufour effect. A brief literature on double diffusive Marangoni convection with diffusion thermal effect for different flows like boundary layer flows, convection flows in a single fluid/porous layer is available. Nagabushanam Reddy et al [7] has investigated numerically the onset of dufour and soret effect on MHD laminar boundary layer flow past a stretching plate by numerical techniques (Shooting and fourth order R-K method) to find various physical parameters. Idowu and Falodun [4] have studied the cross diffusion effects on MHD mass and heat transfer of Walter's – B viscoelastic fluid over a semi-infinite vertical plate. Hamid et al [3] have worked on the problem of thermal diffusion and diffusion thermal effect on thermosolutal Marangoni convection flow of an electrically conducting fluid over a permeable surface. Sumithra. R [10] has considered the composite system to study the double diffusive convection by considering the magnetic field and by using Regular Perturbation technique solve the resulting ODE. Najeeb Alam Khan and Faqiha Sultan [8] have investigated on

**Corresponding author:*

the double diffusive convection flow of Eyring-powell fluid from a cone embedded in porous medium with thermal diffusion and diffusion thermoeffects. They used both numerical techniques and analytical methods are used to solve the nonlinear ordinary differential equations. Krishna Murthy and Vinay Kumar [6] have investigated the MHD forces on double diffusive free convection process along a wavy surface embedded in a doubly stratified fluid-saturated Darcy porous medium under the influence of soret and dufour effect by using Keller Box finite difference scheme to solve the resulting boundary layer equations. Tehreem Nasir et al. [16] have done numerical investigation on MHD flow with soret and dufour effects by using Shooting method to solve the governing equations. They obtained skin friction coefficient, Sherwood and Nusselt numbers for the study through graphs. In a composite system Sumithra et al. [21] has investigate the impact of thermal diffusion effect on Rayleigh Benard(RB) convection with non-Darcy effect and assume that the boundaries are rigid-rigid for composite system. Regular perturbation method is used to solve the ordinary differential equations which are obtained from the corresponding partial differential equations and also the effect of viscosity ratio, Darcy number, diffusivity ratio, solutal Rayleigh number and soret parameters of both fluid and porous layers on critical Rayleigh number has discussed graphically in detail.

DDM convection with Dufour effect has a wide range of its applications in industries especially in crystal growth, semiconductor processing, nuclear waste disposal, hydrology, oceanography, geothermal energy and so on. The occurrence of two-layer system in these applications is common. In this paper, the problem of DDM convection with Dufour effects, in a two-layered component system with incompressible dual component fluid saturated sparsely packed porous layer over which lies a layer of the same fluid, has been studied analytically by Exact Method using Darcy-Brinkmann model The impact of distinct physical parameters on DDM convection has been investigated in detail manner and results are presented graphically. The effect of Dufour parameter, which plays vital role in diffusion-thermal process, when the energy flux due to mass gradient appears, on DDM convection in a two-layered system, has been explored.

2.0 Mathematical modelling

The physical model under the consideration of composite layer system with a horizontal two – component fluid saturated, incompressible, isotropic and sparsely packed porous layer is of thickness d_m and a fluid layer of thickness d. Both the boundaries of the composite layer system are considered to be rigid and these boundaries are acts like heat and mass insulators along with maintained, distinct concentration and temperature. The origin point of the Cartesian coordinate system is taken exactly at the intersection of porous and fluid layer along with direction of z-axis is vertically upwards. In addition to that, for the effect of density variation, Boussinesq approximation is included. Under these following assumptions, the governing equations are, the continuity, momentum, temperature and concentration equations as follows.

For fluid layer,

$$\nabla . \vec{q} = 0 \quad \text{... (1)}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla P + \mu \nabla^2 \vec{q} \quad \text{... (2)}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = k\,\nabla^2 T + k_T\,\nabla^2 c \quad \text{... (3)}$$

$$\frac{\partial c}{\partial t} + (\vec{q}.\nabla)c = k_c \nabla^2 c \quad \text{... (4)}$$

For porous layer,

$$\nabla_m . \vec{q}_m = 0 \quad \text{... (5)}$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{q}_m}{\partial t_m} = -\nabla_m p_m - \frac{\mu}{K}\vec{q}_m + \mu_m \nabla^2 \vec{q}_m \quad \text{... (6)}$$

$$A\frac{\partial T_m}{\partial t_m} + (\vec{q}_m . \nabla_m)T_m = k_m \nabla_m^2 T_m + k_{mT}\nabla_m^2 c_m \quad \text{... (7)}$$

$$\phi\frac{\partial c_m}{\partial t_m} + (\vec{q}_m . \nabla_m)c_m = k_{mc}\nabla_m^2 c_m \quad \text{... (8)}$$

Where $\vec{q}=(u, v, w)$ is the fluid velocity vector, t is the time, μ is the viscosity of fluid, ρ_o is the fluid density, P is the pressure, $A = \frac{(\rho_o c_p)_m}{(\rho c_p)_f}$ is the ratio of heat capacities, c_p is the specific heat at constant pressure, K is the permeability of the sparsely porous medium, T is the temperature, k is the thermal diffusivity, k_c is the solutal diffusivity, k_T is the Dufour coefficient, c is the concentration, ϕ is the porosity, the subscript m refers to the respective physical quantities in porous layer. Consider the basic state, in the composite layer system the fluid layer is stable and also assumed it is inactive, means quiescent. Therefore there is no motion, so, set the velocity vector $\vec{q}$ is zero, where, the mass and heat are transferred only by conduction in the fluid layer, $[u, v, w, p, T, c] = [0,0,0, p_b(z), T_b(z), c_b(z)]$

and in the porous layer, $[u_m, v_m, w_m, p_m, T_m, c_m] =, [0,0,0, p_{mb}(z_m), T_{mb}(z_m), c_{mb}(z_m)]$, the subscript b stands basic state.

For both fluid and porous layer, the temperature distribution and concentration distributions are $T_b(z),\ T_{mb}(z_m)$ and $C_b(z),\ C_{mb}(z_m)$ respectively are found to be

$$T_b(z) = T_0 + \frac{(T_u - T_0)z}{d} \quad \text{in} \quad 0 \le z \le d$$

$$T_{mb}(z_m) = T_0 + \frac{(T_0 - T_L)z_m}{d_m} \quad \text{in} \quad -d_m \le z_m \le 0$$

$$C_b(z) = C_0 + \frac{(C_u - C_0)z}{d} \quad \text{in} \quad 0 \le z \le d$$

$$C_{mb}(z_m) = C_0 + \frac{(C_0 - C_L)z_m}{d_m} \quad \text{in} \quad -d_m \le z_m \le 0$$

Where

$T_0 = \frac{k\, d_m\, T_u + d\, k_m\, T_L}{k\, d_m + d\, k_m} + \frac{d_m k_T\, C_u + k\, T_m\, d\, C_L}{k\, d_m + d\, k_m} - C_0\left(\frac{d\, k\, T_m + d_m\, k_T}{k\, d_m + d\, k_m}\right)$ and $C_0 = \frac{k_s\, C_u d_m + d\, k_{sm} C_L}{d\, k_{sm} + d_m\, k_s}$ are the interface temperature and concentrations respectively. It is evident that the interface temperature depends on interface concentration due to Dufour effects. To analyze the stability of the basic state solution, an infinitesimal perturbations are introduced and the perturbed quantities are of the form.

For fluid layer

$$[\vec{q}, P, T, C\,] = [\,0,\ P_b(z), T_b(z), C_b(z)] + [\vec{q}, P', \theta, S\,]$$

For porous layer

$$[\vec{q}_m, P_m, T_m, C_m\,] = [\,0,\ P_{mb}(z_m),\ T_{mb}(z_m), C_{mb}(z_m)] + [\vec{q}_m, P'_m, \theta_m, S_m]$$

Substitute above perturbations into equations (1) - (8) and apply two times curl on the momentum equations to eliminate the pressure term. The resulting equations are linearized for (1) - (8), then they are non-dimensionalised using the following variables $\frac{d^2}{k},\ \frac{k}{d},\ C_0 - C_U,\ d$ and $T_0 - T_U$ for time, velocity, concentration, length and temperature respectively in region-1 and $\frac{d_m^2}{k_m},\ \frac{k_m}{d_m},\ C_L - C_0,\ d_m$ and $T_L - T_0$ are the corresponding scale factors in region-2. Applying normal mode expansions to the perturbed non-dimensionalised quantities (following Sumithra [10]) result in the following ordinary differential equations,

In $0 \le z \le 1$

$$(D^2 - a^2)^2 W = 0 \qquad \text{... (9)}$$

$$(D^2 - a^2)\Theta + W + D_r(D^2 - a^2)S = 0 \qquad \text{... (10)}$$

$$\tau(D^2 - a^2)S + W = 0 \qquad \text{... (11)}$$

In $-1 \le Z_m \le 0$

$$[\hat{\mu}\beta^2(D_m^2 - a_m^2) - 1](D_m^2 - a_m^2)W_m = 0 \qquad \text{... (12)}$$

$$(D_m^2 - a_m^2)\Theta_m + W_m + D_{mr}(D_m^2 - a_m^2)S_m = 0 \qquad \text{... (13)}$$

$$\tau_{pm}(D_m^2 - a_m^2)S_m + W_m = 0 \qquad \text{... (14)}$$

Where, for fluid layer, $p_r = \frac{\nu}{k}$ is the Prandtl number, $\tau = \frac{k_c}{k}$ is the ratio of salinity diffusivity to thermal diffusivity, $\nu = \frac{\mu}{\rho_0}$ is the kinematic viscosity and $D_r = \frac{k_T\,(C_0 - C_u)}{k\,(T_0 - T_u)}$ is Dufour coefficient. For the porous layer, $p_{rm} = \frac{\phi\nu}{k_m}$ is the Prandtl number, $\beta^2 = \frac{K}{d_m^2} = D_a$ is the Darcy number, $\tau_{pm} = \frac{k_{mc}}{k_m}$ is the ratio of salinity diffusivity to thermal diffusivity, $D_{mr} = \frac{k_{mT}\,(C_L - C_0)}{k_m\,(T_L - T_0)}$ is the Dufour coefficient and $\hat{\mu} = \frac{\mu_m}{\mu}$ is the viscosity ratio.

Boundary conditions

At the upper boundary.

$$W(1) = 0, \quad D^2W(1) - Ma^2\Theta(1) - M_s a^2 S(1) = 0, \quad D\Theta(1) = 0, \quad DS(1) = 0$$

At the interface

$$W(0) = \frac{\varsigma}{\varepsilon_T} W_m(0), \qquad S = \frac{\varepsilon_s}{\varsigma} S_m, \qquad \Theta(0) = \frac{\varepsilon_T}{\varsigma}\Theta_m(0)$$

$$DW(0) = \frac{\varsigma^2}{\varepsilon_T} D_m W_m(0), \qquad DS(0) = D_m S_m(0), \qquad D\Theta(0) = D_m\Theta_m(0)$$

$$[D^3 - 3a^2 D]W(0) = \frac{-\varsigma^2}{D_a \varepsilon_T} D_m W_m(0) + \frac{\varsigma^4}{\varepsilon_T}[D_m^3 3a_m^2 D_m]W_m(0)$$

At the lower boundary

$$W_m(-1) = 0, \qquad D_m W_m(-1) = 0, \quad D_m\Theta_m(-1) = 0, \quad D_m S_m(-1) = 0$$

Where $M = -\frac{\partial\sigma}{\partial T}\frac{(T_0 - T_u)d}{\mu_k}$, $M_s = -\frac{\partial\sigma}{\partial s}\frac{(C_0 - C_u)d}{\mu_k}$ are the thermal and solute Marangoni numbers respectively. $\varepsilon_T = \frac{k}{k_c}$, $\varepsilon_s = \frac{k_m}{k_{mc}}$ are the ratios of thermal diffusivity to solutal diffusivity for both fluid and porous layers respectively and $\varsigma = \frac{d}{d_m}$ is the depth ratio.

Solution by Exact Method

The final solutions of the differential equations (9) and (12) are not depend on $\Theta(z)$, $S(z)$, $\Theta_m(Z_m)$, $S_m\,(Z_m)$. The expressions for W and W_m are solved by using the velocity boundary conditions and are obtained as below,

$$W(z) = A_1\,[cosh(az) + \Delta_6\, z\cosh(az) + \Delta_5 \sinh(az) + \Delta_7\, z\sinh(az)]$$

And

$$W_m(z_m) = \Delta_4 \cosh(a_m z_m) + \Delta_2 \sinh(a_m z_m) + \Delta_3 \cosh(\delta z_m) + \Delta_1 \sinh(\delta z_m)$$

Where

$$\Delta_1 = \frac{\lambda_{28}\,\lambda_{29} - \lambda_{31}\,\lambda_{26}}{\lambda_{27}\,\lambda_{29} - \lambda_{30}\,\lambda_{26}}\ , \qquad \Delta_2 = \frac{\lambda_{28} - \lambda_{27}\,\Delta_1}{\lambda_{26}}$$

$$\Delta_3 = \frac{-\cosh a_m \left(\frac{\varepsilon_s}{\varsigma}\right) + \sinh\delta\,\Delta_1 + \sinh a_m\,\Delta_2}{\cosh\delta - \cosh a_m}$$

$$\Delta_4 = \frac{\varepsilon_T}{\varsigma} - \frac{\Delta_1 \sinh\delta + \Delta_2 \sinh a_m - \cosh a_m \left(\frac{\varepsilon_T}{\varsigma}\right)}{\mathrm{Cosh}\delta - \mathrm{Cosh} a_m}$$

$$\Delta_6 = (\lambda_3 - a\,\lambda_1)\Delta_2 + (\lambda_4 - a\,\lambda_2)\Delta_1, \qquad \Delta_7 = (\lambda_7 - \lambda_6)\Delta_3 + \lambda_6\,\lambda_5 - a$$

$$\Delta_5 = \left(\frac{\varsigma^2 a_m}{2a^3 D_a \varepsilon_T} + \frac{\varsigma^4 a_m^3}{\varepsilon_T a^3}\right)\Delta_2 + \left(\frac{\varsigma^2\delta}{2a^3 D_a \varepsilon_T} - \frac{\varsigma^4(\delta^3 - 3a_m^2\delta)}{2a^3\varepsilon_T}\right)\Delta_1$$

$$\lambda_{28} = \frac{\varepsilon_T}{\varsigma}(\delta \sinh\delta \cosh a_m - a_m \cosh\delta \sinh a_m)$$

$$\lambda_{29} = (A\lambda_7 - A\lambda_6)a_m \cosh a_m - (A\lambda_1 + \lambda_3 - a\lambda_1)(-\delta \sinh\delta + a_m \sinh a_m)$$

$$\lambda_{31} = -a_m \sinh a_m (\lambda_5 \lambda_7 \tanh a + \lambda_6 \lambda_5 \tanh a + \lambda_6 \lambda_5 - a - 1) - \delta \sinh\delta(1 + a - \lambda_6\lambda_5)$$

$$\lambda_{26} = \delta \sinh\delta \sinh a_m - a_m(\sinh a_m)^2 - a_m \cosh\delta \cosh a_m + a_m(\cosh a_m)^2$$

$$\lambda_{27} = \delta(\sinh\delta)^2 - a_m \sinh a_m \sinh\delta - \delta(\cosh\delta)^2 + \delta \cosh\delta \cosh a_m$$

$$\lambda_{30} = \delta \cosh\delta(A\lambda_7 - A\lambda_6) - (A\lambda_2 + \lambda_4 - a\lambda_2)(-\delta \sinh\delta + a_m \sinh a_m)$$

$$A = \tanh a, \qquad \lambda_1 = \frac{\varsigma^2 a_m}{2a^3 D_a \varepsilon_T} + \frac{\varsigma^4 a_m^3}{\varepsilon_T a^3}, \qquad \lambda_2 = \frac{\delta\varsigma^2}{2a^3 D_a \varepsilon_T} - \frac{\varsigma^4(\delta^3 - 3\delta a_m^2)}{2\varepsilon_T a^3}$$

$$\lambda_4 = \frac{\delta\varsigma^2}{\varepsilon_T}, \qquad \lambda_5 = \frac{\varepsilon_T}{\varsigma}, \qquad \lambda_6 = \frac{\mu_m \varsigma^3 a_m^2}{\mu a \varepsilon_T}$$

$$\lambda_7 = \frac{\mu_m \varsigma^3 \delta^2}{2a\mu\varepsilon_T}, \qquad \lambda_3 = \left(\frac{\varsigma^2}{\varepsilon_T}\right) a_m, \qquad \delta = \sqrt{a_m^2 + \frac{1}{\hat{\mu} D_a}}$$

The concentration distributions $S(z)$ and $S_m(Z_m)$ are getting from the Differential Equations (11) and (14) by substituting the algebraic expressions for $W(z)$ and $W_m(z_m)$ and are solved using the concentration boundary conditions and are as below.

$$S(z) = A_1 \left[\Delta_{10} \cosh(az) + \Delta_{11} \sinh(az) - \frac{f(z)}{\tau}\right]$$

And

$$S_m(z_m) = A_1\left[\Delta_8 \cosh(a_m z_m) + \Delta_9 \sinh(a_m z_m) - g(z)\tau_{pm}{}^{-1}\right]$$

Where

$$f(z) = \frac{z\sinh(az)}{2\,a} + \frac{\Delta_5 z\cosh(az)}{2a} + \delta_1(z) + \delta_2(z)$$

$$\delta_1(z) = \frac{\Delta_6}{4a}\left[z^2\sinh(az) - \frac{z\cosh(az)}{a}\right]$$

$$\delta_2(z) = \frac{\Delta_7}{4a}\left[z^2\cosh(az) - \frac{z\sinh(az)}{a}\right]$$

$$g(z) = \frac{\Delta_4 z_m \sinh(a_m z_m)}{2a_m} + \frac{\Delta_2 z_m \cosh(a_m z_m)}{2a_m} + \delta_3$$

$$\delta_3 = \left(\frac{\Delta_3}{\delta^2 - a_m^2}\right)\cosh(\delta z_m) + \left(\frac{\Delta_1}{\delta^2 - a_m^2}\right)\sinh(\delta z_m)$$

$$\Delta_8 = \frac{p_1 \cosh a + \delta_4 - a p_2}{-a_m \cosh a \tanh a_m - \left(\frac{a}{\varsigma}\right)\varepsilon_s \sinh a}, \qquad \Delta_9 = \frac{g_3}{a_m \cosh a_m \tau_{pm}} + \Delta_8 \tanh a_m$$

$$\Delta_{10} = \frac{\Delta_8 \varepsilon_s}{\varsigma} - \frac{\Delta_3 \varepsilon_s}{\varsigma \tau_{pm}(\delta^2 - a_m^2)}, \qquad \Delta_{11} = \frac{p_2}{\cosh a} - \left(\frac{\varepsilon_s}{\varsigma}\right)\Delta_8 \tanh a$$

$$\delta_4 = \left(\frac{g_3 \cosh a}{\tau_{pm} \cosh a_m}\right), \qquad p_2 = \frac{f_1}{a\tau} + \frac{\Delta_3 \varepsilon_s \sinh a}{\varsigma \tau_{pm}(\delta^2 - a_m^2)}$$

$$p_1 = \frac{1}{\tau}\left[\frac{\Delta_5}{2a} - \frac{\Delta_6}{4a^2}\right] - \frac{1}{\tau_{pm}}\left[\frac{\Delta_2}{2a_m} + \frac{\delta \Delta_1}{\delta^2 - a_m^2}\right], \qquad g_3 = \delta_7 + \delta_8 - \delta_9 + \delta_{10}$$

$$\delta_7 = \frac{\Delta_4}{2a_m}(-a_m \cosh a_m - \sinh a_m), \qquad \delta_8 = \frac{\Delta_2}{2a_m}(a_m \sinh a_m + \cosh a_m)$$

$$\delta_9 = \sinh\delta \left(\frac{\Delta_3 \delta}{\delta^2 - a_m^2}\right), \qquad \delta_{10} = \cosh\delta \left(\frac{\Delta_1 \delta}{\delta^2 - a_m^2}\right)$$

$$f_1 = \frac{\delta_{11}}{2a} + \frac{\delta_{12}\,\Delta_5}{2a} + \frac{\Delta_6}{4a}\left(\delta_{11} - \frac{\cosh a}{a}\right) + \frac{\Delta_7}{4a}\left(\delta_{12} - \frac{\sinh a}{a}\right)$$

$$\delta_{11} = a\, cosha + \sinh a, \qquad \delta_{12} = a \sinh a + \cosh a$$

The temperature distributions $\Theta(z)$ and $\Theta m(z_m)$ are getting from the Differential Equations (10) and (13) by substituting the algebraic expressions $W(z)$, $W_m(z_m)$, $S(z)$ and $S_m(z_m)$ and are solved by the make use of temperature boundary conditions and are as follows.

$$\Theta(z) = \Delta_{16}\, coh(az) + \Delta_{13} \sinh(az) + \delta_{17}(z) + \delta_{18}(z)$$

And

$$\Theta(z_m) = \Delta_{15} \cosh(a_m z_m) + \Delta_{14} \sinh(a_m z_m) + \delta_{19}(z) + \delta_{20}(z) + \delta_{21}(z)$$

Where

$$\delta_{17}(z) = \delta_{13}\Delta_7\, z\, sinhaz + \delta_{13}\Delta_6\, z\, coshaz$$

$$\delta_{18}(z) = \delta_{15}\Delta_7\, z^2\, coshaz + \delta_{15}\Delta_6\, z^2\, sinhaz$$

$$\delta_{19}(z) = \left[\frac{D_{mr}\Delta_4}{2a_m \tau_{pm}} - \frac{\Delta_4}{2a_m}\right] z_m \sinh(a_m z_m) + \left[\frac{D_{mr}\Delta_2}{2a_m \tau_{pm}} - \frac{\Delta_2}{2a_m}\right] z_m \cosh(a_m z_m)$$

$$\delta_{20}(z) = \left[\frac{-\Delta_1}{\delta^2 - a_m^2} - \frac{\Delta_1 \delta^2 D_{mr}}{(\delta^2 - a_m^2)^2} - \frac{\Delta_1 a_m^2 D_{mr}}{\tau_{pm}(\delta^2 - a_m^2)^2}\right] \sinh(\delta z_m)$$

$$\delta_{21}(z) = \left[\frac{-\Delta_3}{\delta^2 - a_m^2} - \frac{D_{mr}\Delta_3 \delta^2}{(\delta^2 - a_m^2)^2} - \frac{a_m^2 \Delta_3 D_{mr}}{\tau_{pm}(\delta^2 - a_m^2)^2}\right] \cosh(\delta z_m)$$

$$\delta_{13} = \left(\frac{D_r}{2a\tau} - \frac{1}{2a}\right) + \left(\frac{1}{4a^2} - \frac{3D_r}{16a^2\tau}\right), \quad \delta_{15} = \frac{3D_r}{4\tau} - 1$$

$$\Delta_{13} = \frac{t_4 \sinh a_m + t_3 \sinh a \lambda_5 + t_2 \sinh a\ \delta_{21}}{\cosh a \sinh a_m + \sinh a \cosh a_m \delta_{23}}$$

$$\delta_{24} = \left(\frac{a}{a_m}\right)\Delta_{13} - \frac{\Delta_2}{a_m}\left(\frac{D_{mr}}{2a_m \tau_{pm}} - \frac{1}{2a_m}\right), \quad \delta_{25} = \left(\frac{1}{4a^2} - \frac{3D_r}{16a^2\tau}\right)\Delta_6$$

$$\delta_{26} = \frac{\delta \Delta_1}{a_m(\delta^2 - a_m^2)}\left(-1 - \frac{\delta^2 D_{mr}}{\delta^2 - a_m^2} - \frac{a_m^2 D_{mr}}{\tau_{pm}(\delta^2 - a_m^2)}\right)$$

$$\Delta_{15} = t_4\left(\frac{\varsigma}{\varepsilon_T \sinh a}\right) - \Delta_{13} \coth a\left(\frac{\varsigma}{\varepsilon_T}\right), \quad \Delta_{14} = \delta_{24} + \delta_{25} - \delta_{26}$$

$$\Delta_{16} = \Delta_{15}\left(\frac{\varepsilon_T}{\varsigma}\right) + \left(\frac{\varepsilon_T}{\varsigma}\right)\left[-1 - \frac{D_{mr}\delta^2}{\delta^2 - a_m^2} - \frac{a_m^2 D_{mr}}{\tau_{pm}(\delta^2 - a_m^2)}\right]\frac{\Delta_3}{\delta^2 - a_m^2}$$

$$\delta_{22} = \frac{\varepsilon_T \cosh a_m}{\varsigma a_m}, \qquad \delta_{23} = \frac{\varepsilon_T a}{a_m \varsigma}, \qquad t_1 = \lambda_{31} + \lambda_{32}$$

$$\lambda_{31} = \frac{\cosh a}{a}[\delta_{27} + \delta_{28}\Delta_7 + \Delta_5\delta_{29} + \Delta_6\delta_{30}]$$

$$\lambda_{32} = \frac{\sinh a}{a}[\delta_{27}\Delta_5 + \delta_{28}\Delta_6 + \delta_{29} + \Delta_7\delta_{30}]$$

$$\delta_{27} = \frac{1}{2} - \frac{D_r}{\tau}, \qquad \delta_{28} = \frac{3D_r}{16a\tau} - \frac{1}{4a} - \frac{3D_r}{8a\tau} + \frac{1}{2a}$$

$$\delta_{29} = \frac{1}{2a} - \frac{D_r}{2a\tau}, \qquad \delta_{30} = \frac{3D_r}{16a^2\tau} - \frac{1}{4a^2} - \frac{3D_r}{16\tau} + \frac{1}{4}$$

$$t_2 = \delta_{31} - \Delta_5\left(\frac{D_r}{2a\tau} - \frac{1}{2a}\right) - \Delta_6\left(\frac{1}{4a^2} - \frac{3D_r}{16a^2\tau}\right)$$

$$\delta_{31} = \Delta_2\left(\frac{D_{mr}}{2a_m\tau_{pm}} - \frac{1}{2a_m}\right) + \frac{\delta\Delta_1}{\delta^2 - a_m^2}\left(-1 - \frac{\delta^2 D_{mr}}{\delta^2 - a_m^2} - \frac{a_m^2 D_{mr}}{\tau_{pm}(\delta^2 - a_m^2)}\right)$$

$$t_3 = (a_m \cosh a_m + \sinh a_m)\,\delta_{34}\Delta_4 - \delta_{33} + \delta_{32}$$

$$\delta_{32} = \delta_{35}\left(\frac{\Delta_3\delta \sinh\delta}{a_m(\delta^2 - a_m^2)} - \frac{\Delta_1\delta \cosh\delta}{a_m(\delta^2 - a_m^2)}\right), \qquad t_4 = t_1 - \delta_{35}\left[\frac{\Delta_3\varepsilon_T \mathrm{Sinh}a}{\varsigma(\delta^2 - a_m^2)}\right]$$

$$\delta_{33} = (a_m \sinh a_m + \cosh a_m)\delta_{34}\Delta_2\ , \qquad \delta_{34} = \frac{D_{mr}}{2a_m^2\tau_{pm}} - \frac{1}{2a_m^2},$$

$$\delta_{35} = -1 - \frac{D_{mr}\delta^2}{\delta^2 - a_m^2} - \frac{a_m^2 D_{mr}}{\tau_{pm}(\delta^2 - a_m^2)}$$

To obtain the Eigenvalue, the thermal Marangoni number

The thermal Marangoni number is obtained by using the coupled boundary condition at the upper fluid boundary which is free with Marangoni effects, which is

$$D^2W(1) - Ma^2\Theta(1) - M_s a^2 S(1) = 0$$

By solving for M, by substituting all the known quantities, we get

$$M = \frac{\cosh a\,\lambda_{33} + \sinh a\,\lambda_{34} - M_s\,a^2\left(\Delta_{10}\cosh a + \Delta_{11}\sinh a - \frac{1}{\tau}[\lambda_{35} + \lambda_{36} + \lambda_{37}]\right)}{\Delta_{16}\,\mathrm{Cosh}a + \Delta_{13}\,\mathrm{Sinh}a + \lambda_{38} + \lambda_{39} + \lambda_{40}}$$

Where

$$\lambda_{33} = a^2 + 2\,a\,\Delta_7 + a^2\Delta_6, \qquad \lambda_{34} = a^2\Delta_5 + 2\,a\,\Delta_6 + a^2\,\Delta_7$$

$$\lambda_{35} = \frac{sinha}{2\,a} + cosha\,\frac{\Delta_5}{2\,a}, \qquad \lambda_{36} = (sinha - \frac{cosha}{a})\frac{\Delta_6}{4\,a}$$

$$\lambda_{37} = (sinha - \frac{cosha}{a})\frac{\Delta_6}{4\,a}$$

$$\lambda_{38} = \text{sinha}\left[\left(\frac{D_r}{2a\tau} - \frac{1}{2a}\right) + \left(\frac{1}{4a^2} - \frac{3D_r}{16a^2\tau}\right)\Delta_7\right]$$

$$\lambda_{39} = \text{cosha}\left[\left(\frac{D_r}{2a\tau} - \frac{1}{2a}\right)\Delta_5 + \left(\frac{1}{4a^2} - \frac{3D_r}{16a^2\tau}\right)\Delta_6\right]$$

$$\lambda_{40} = \frac{\Delta_7\,\text{cosha}}{4\,a}\left(\frac{3D_r}{4\tau} - 1\right) + \frac{\Delta_6\,\text{sinha}}{4\,a}\left(\frac{3D_r}{4\tau} - 1\right)$$

Graphical Interpretations

The thermal Marangoni number M which is the eigenvalue of the problem Darcy-Brinkmann DDM convection in a two-layered system with Dufour effects is obtained in closed form as an expression of physical quantities the Darcy number, the ratio of diffusivities, the viscosity ratio, the Dufour parameters and the depth ratio. To illustrate the impacts of these parameters on the stability of the system, the eigenvalue which is the thermal Marangoni number in DDM convection problem, is plotted as a function of depth ratio for various of values of other physical parameters.

The impact of porous parameter Darcy number Da on M for fixed values of a =1.5, ε_T=0.5, $\hat{\mu}$ =1, M_s=10, Dr=3, D_{mr}=1, ε_s=0.25 is shown in fig-1 for D_a= 0.1, 0.2, 0.3. From the figure, clearly evident that one of the interesting fact the curves are converging at both the ends. Fixed a fixed value of the depth ratio (ς), the value of M falls down with increase the value of porous parameter Darcy number D_a, the DDM convection is preponed for larger values of Darcy number as there is more window for the fluid to move in the porous layer, hence the two-layer system is destabilized.

The effect of ratio of thermal-solutal diffusivities in the porous layer, ε_s on thermal Marangoni number M for fixed values of a=1, ε_T=0.75, D_a=0.1, $\hat{\mu}$ =1, M_s=10, D_r=3, D_{mr}=2 is shown in fig-2 for ε_s= 0.25, 0.5, 0.75. For values of depth ratio ς in the range [0,0.6], the thermal Marangoni number decreases with increase in the ratio of thermal-solutal diffusivities in the porous layer and for ς > 0.6 variation in ε_s has no much effect on the same. Hence this parameter is effective for minor values of ς, that is for sparsely packed porous layer dominant two-layer system, which is physically sensible.

The effect of the ratio of thermal-solutal diffusivities in the fluid layer, ε_T on M for fixed values D_a=0.1, a=1, ε_s=0.75, $\hat{\mu}$ =1, M_s=10, D_r=3, D_{mr}=2 is shown in fig-3 for ε_T= 0.25, 0.5, 0.75. There is a dual behavior of the ratio of thermal-solutal diffusivities in the fluid layer, ε_T on the DDM convection depending on the depth ratio. For the depth ratio, ς values in the range [0,0.6], the value of M rises up with increase in this ratio and for ς > 0.6, the value of M decreases with increase in the same. Hence for porous layer dominant two-layered systems, the higher values of this ratio and for fluid layer dominant two-layer systems, the smaller values of the ratio, are convenient to control DDM convection in a two-layered system.

The effect $\hat{\mu}$ on M for fixed values D_a=0.1, a=1, ε_T=0.25, ε_s=0.25, M_s=10, D_r=1, D_{mr}=1 is shown in fig-4 for $\hat{\mu}$ =1, 1.2, 1.5. From the figure, it is evident that this influence of the viscosity ratio is effective for only smaller values of depth ratio, that is, for porous layer dominant two-layer system. Which is quite reasonable the effective viscosity of the fluid in the porous layer is dominant only for PLD two-layer system. For a fixed value of depth ratio, thermal Marangoni number increases with increasing the values of μ. Hence, larger values of the effective viscosity of the fluid in the sparsely packed porous layer makes the two-layer system stable by postponing the DDM convection.

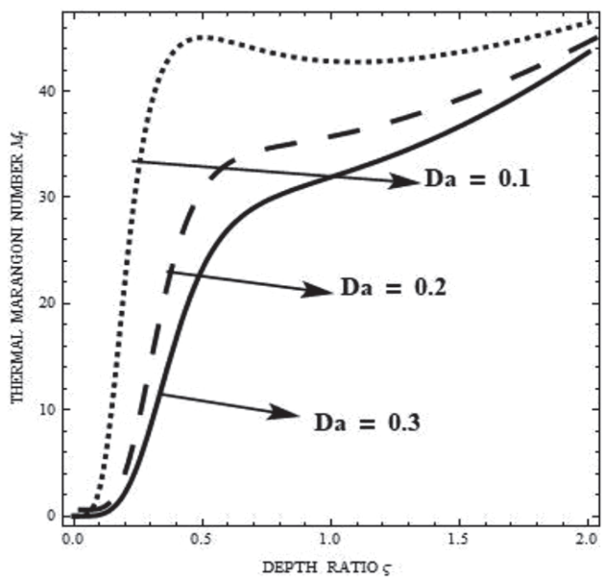

Figure 1

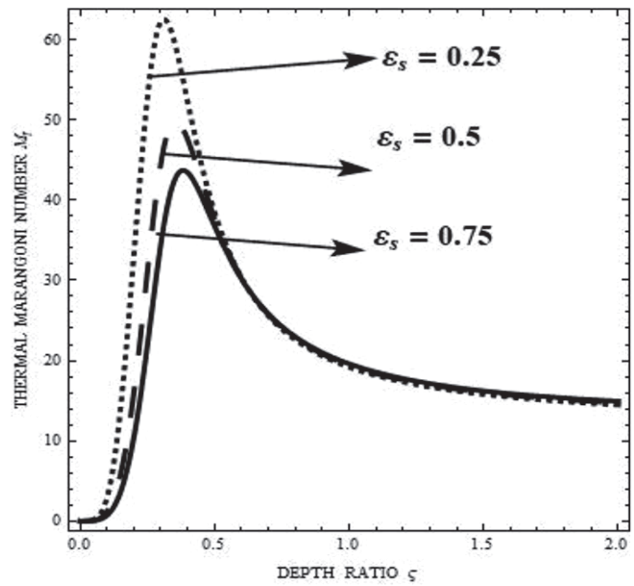

Figure 2

Figure (1 and 2) shows the effect of D_a and ε_s on Thermal Marangoni number M

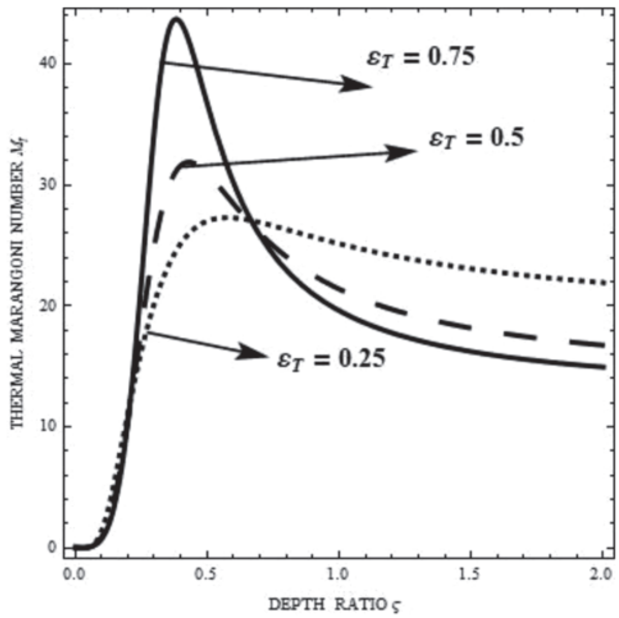

Figure 3

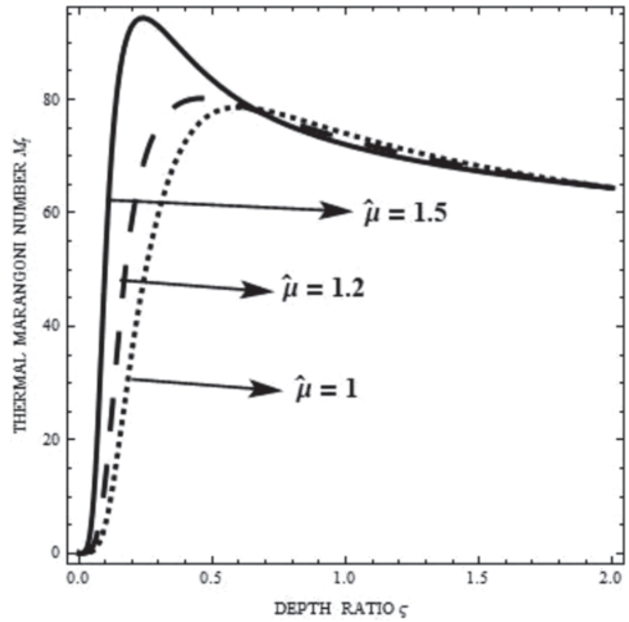

Figure 4

Figure (3 and 4) shows the effect of ε_T and $\hat{\mu}$ on Thermal Marangoni number M

The impact of Dufour parameter D_r of the fluid layer, on the thermal Marangoni number M for fixed values D_a=0.1, a=1, ε_T=0.25, ε_s=0.75, μ=1.5, Ms=10, D_{mr}=1 is shown in fig-5, for Dr=1,2,3 . From the figure, clearly evident that one of the interesting fact, the curves are diverging at one of the end indicating that the impact of Dufour parameter inside the fluid layer is effective for the bigger values of depth ratio, that is, for fluid layer dominant two-layered system as expected. For a set value of depth ratio, the value of dufour parameter rises up, decreases the values of M and the decrease is drastic, hence the thermal Marangoni number is very sensitive to Dufour effects in the fluid layer. The smaller values of this parameter is convenient to control DDM convection in a fluid layer dominant two-layer system.

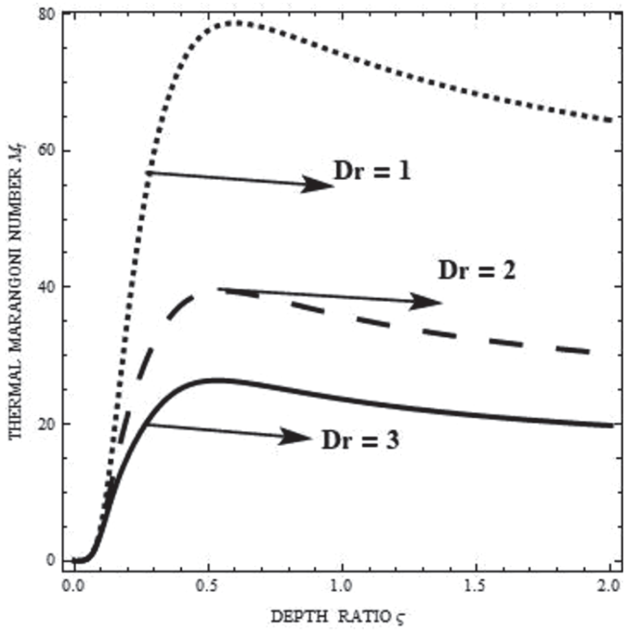

Figure 5

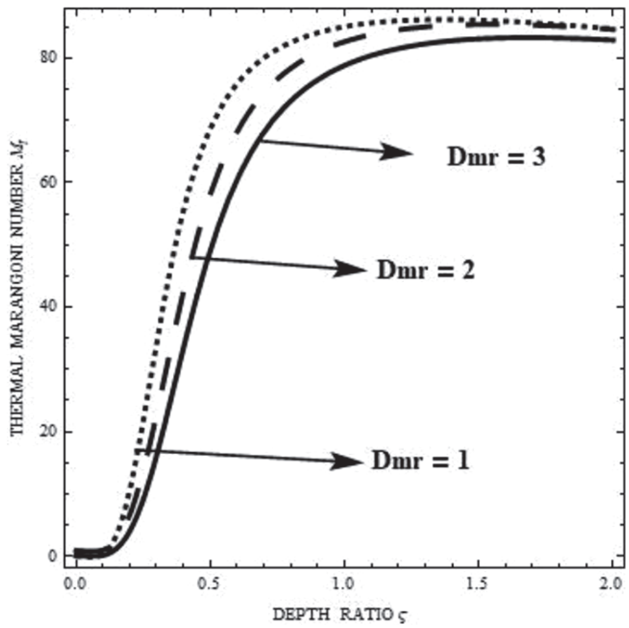

Figure 6

Figure (5 and 6) shows the effect of *Dr* and *Dmr* on Thermal Marangoni number M

The impact of Dufour parameter D_{mr} of the porous layer on *M*, for the fixed values of D_a=0.1, *a*=1, ε_T=0.25, ε_s=0.75, $\hat{\mu}$ =1.5, M_s=10, *Dr*=1 is shown in fig-6, for D_{mr}=1, 2, 3. From the figure, it is clear the curves are converging at both the ends, indicating that the effect of Dufour parameter of the porous layer is effective for the moderate values of depth ratio. For a fixed value of depth ratio, the increase in the dufour parameter of the porous layer, decreases the thermal Marangoni number slightly and this small change is effective only the range of depth ratio, shown in the figure 6.

Conclusions

The problem of Darcy-Brinkmann DDM convection in a two-layered system with Dufour effects is solved in closed form and the following deductions are made from the study.

1. The cross-diffusion effects like dufour effects plays a significant role on the onset of DDM convection. In the fluid layer, Dufour parameter has a crucial impact on DDM convection in the two-layered system.
2. Larger values of the Dufour parameters of both the layers support the stability of the two-layer system and control DDM convection.
3. The thermal-solutal diffusivity ratios in the fluid and porous layers do contribute to the DDM convection. Larger values of thermal-solutal diffusivity ratio in the fluid layer and lower values of thermal-solutal diffusivity ratio in the porous layer support stability for porous layer dominant two-layered system.
4. Larger values of the Darcy number destabilize the system, hence DDM convection can be preponed
5. Higher the viscosity ratio, the more stable is the system and DDM convection is postponed

Acknowledgements

The first author R Sumithra expresses her heart felt thanks to VGST, KSTEPS, Bengaluru, Karnataka, India, for sponsoring this investigation under K–FIST L2, Two years Research Project Scheme, of the Academic year 2020-21.

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