MANAS KUMAR MALLICK
B S CHOUDHARY
and
G BUDI

Inverse square distance weighting vis-à-vis ordinary kriging techniques in resource estimation

Geostatistics play an important role for reserve estimation in mining industry. Geostatistical tools became popular because of its high degree of accuracy and time saving process for estimation. The uncertainty of geological deposit can be populated by geo-statistical tools. The limestone ore deposit is studied in this paper. The assay value of individual constituents of limestone ore i.e CaO, SiO$_2$, Al$_2$O$_3$ and Fe$_2$O$_3$ are determined for a block by using inverse square distance weighting (ISDW) method. The average assay value of those individual constituents are 45.85, 15.94, 1.56 and 0.82 percentage respectively. The assay value of CaO is also estimated by two linear method of estimation i.e ISDW and ordinary kriging (OK). The assay value of CaO are determined by 45.85 and 44.67 percentage respectively. The assay values are properly validated and concluded accordingly. The application of ISDW and OK are implemented to build the resource model together in order to assess the uncertainty of the deposit. Grade estimation by using different geo-statistical techniques are done by SURPAC mine planning software.

Keywords: SURPAC software, ordinary kriging, inverse square distance weighting, geostatistics, ore reserve estimation.

1. Introduction

Mine planning is necessary for determination of total reserve and calculation of average grade value for a particular deposit in order to make it reality. Planning is required for quantifying the resources and bring them by giving some weightage. From the planned data the management can schedule the particular zone where to mine out the mineral at a particular time. Sometime high grade ore is asked by the management for blending in order to fulfill the demand of market.

A 3 dimensional mining model is developed under a block model and the whole process is known as block modelling. The objective of block modelling is to characterize the whole

Messrs. Manas Kumar Mallick, Department of Mining Engineering, BIT Sindri Dhanbad, B S Choudhary and G Budi, Department of Mining Engineering, IIT (ISM) Dhanbad. corresponding author e-mail: mallick.manas2@gmail.com

area into so many blocks but under unidentified mineralization zone (e.g. Isaaks et al.,1989). Open pit mine planning is done by first generating the solid representative block model from which the average assay value cab be determined. The block model is subdivided into smaller blocks and sub blocks for better valuation. The block model is so framed that the solid ore can be accommodated within that block.

In block model each unit block contains geological attributes i.e grades of different minerals, specific gravity, partial percentage, rock code, lithology which is predefined before framing a block model. After geological block modelling, the conversion of block economic model by incorporating financial parameters such as mining and process cost, commodity price etc. Block economic model which gives a platform for mine planning and design process. For the estimation process the source of information mostly comes from a restricted number of exploratory boreholes. Due to unavailability of data the whole blocks are not contained the grade value. In resource estimation the value of unsampled blocks are determined from nearby boreholes.

In the field of mine planning and designing several works have been done by various researchers using this software. The mismatch of grade-tonnage and variation in estimation of blocks can be minimized with the help of geo-statistics (Haldar, 2013). Orebody modelling and comparative study of different geo-statistical tools for reserve estimation has been studied (e.g. Moharaj, et al 2014).

The distance between different sample point to the estimating points are considered in inverse square distance weighting technique in geo-statistical analysis. The assumption is made on the basis of, sampled points which have the same distance to the unsampled point, will show the equal resemblance. The resemblance of assay value depends on the distance between the two sample points. This method of estimation depends upon the weight of the sample point as well as the distance between the two points to be calculated. The inverse square distance weighting technique is coming under the deterministic method of interpolator. The method of estimation is very simple and easy to implement in mine planning sector.

OK method is a probabilistic method of valuation in order

to figure out the uncertainty of ore deposit. Kriging estimator unites the co-variance of different points in a sample and co-variance between the sample location and prediction point. Kriging estimator minimizes the prediction error or error variance. So the quantity known as mean square prediction error (Fig.1).
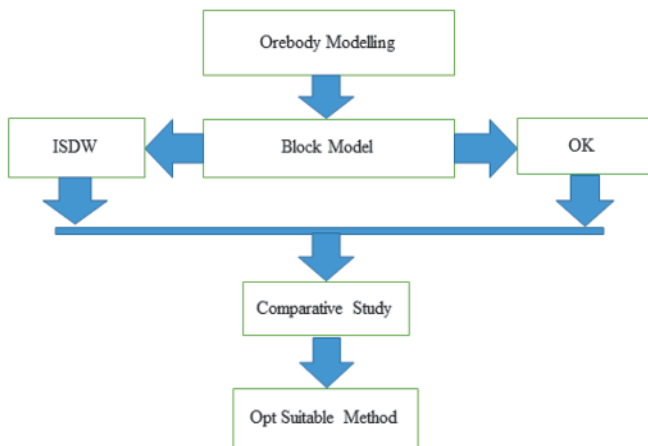

Fig.1 Illustration of the application of estimation

An attempt is made for relative study of linear geo-statistical techniques for resource estimation of a limestone ore deposit in this paper. Both inverse square distance weighting and ordinary kriging estimators are used for assessment of whole ore block after assigning attributes and constraints as well.

2. Methodology

The borehole data of a limestone deposit is used as the main input data to showcase how this software is applied in reserve estimation for both geostatistical methods. The drillhole database is created and those drillholes are shown in a contour plan (Fig.3). Down the drill holes the assay value of CaO is indicated in order to classify them (i.e high grade, low grade and waste) along the drillhole depth with different colours (Fig.2). The assay value of every section along the length of the drillhole is shown with unique number for representing the grade of that particular section.
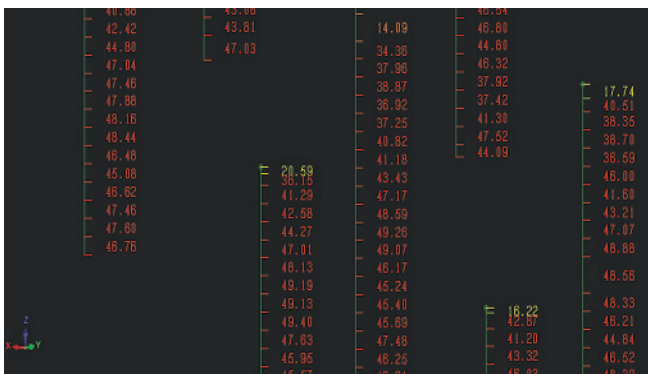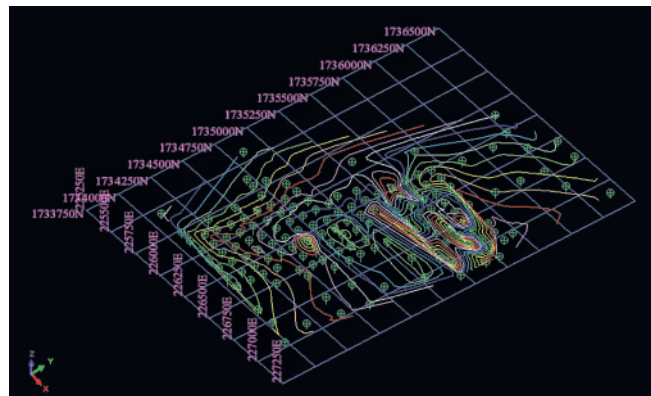

Fig.2 Illustration of drillholes labelling

Fig.3 Illustration of drillholes in contour plan

In the process of electronic ore reserve estimation, total 156 numbers of exploratory borehole data from a limestone ore deposit is used. Geological database is generated by using exploratory borehole data. The assay values are composited within the interval of 1.5 m down the hole with minimum 75% utilization of sample. Composited assay values are taken for development of geostatistical model. Finally, estimation is done by using both linear geostatistical techniques for getting the mean assay value of the deposit. The statistical data of limestone ore shows the frequency of population in the data set for initial knowhow about the deposit.

SURPAC software is used for solid ore modelling as well as block modelling in this work. For better grade determination block model is normally developed because the whole focused area is divided into smaller blocks. The smaller blocks are investigated systematically for getting good resource estimation.

2.1. Basic statistics

In every 2 m run length down the hole, raw data has been composited. From the composited assay value, a statistical report (Table.1) is generated for analysis purpose. Several frequency of grade values is found from the histogram (Fig.4). The grade values are of negatively skewed.

Twenty number of histogram bars are drawn for the frequency analysis of assay value. Statistical analysis is required to provide the idea about the presence of outlier as well as bi-modal distribution in the data set. Outlier is removed from the data set for variogram analysis in order to minimize the error in estimation.

2.2. Solid ore modelling

The solid modelling or 3-D modelling is the visualization of the extent of orebody that is captured within the boundary limit. The individual segment of mineralization zone is triangulated in order to form the solid orebody by sectioning followed by digitizing. The segments are undergone different cleaning test (i.e cross-over, duplicate points, spikes) before forming a solid orebody. After completion of the test, the
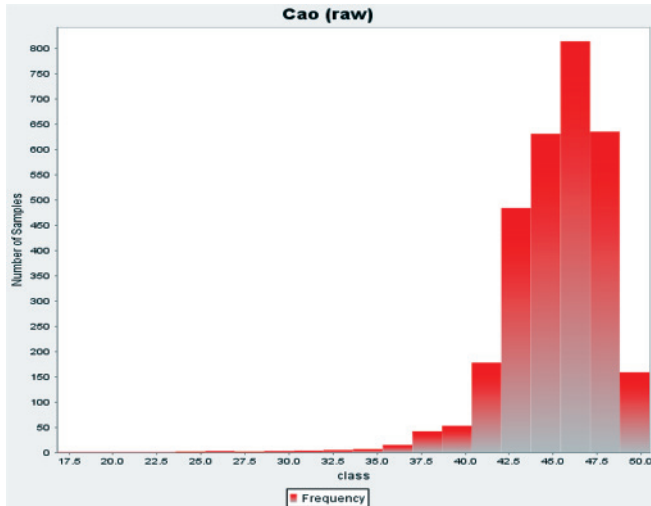
Fig.4 Histogram of raw data of limestone ore

TABLE 1: STATISTICAL REPORT OF RAW DATA

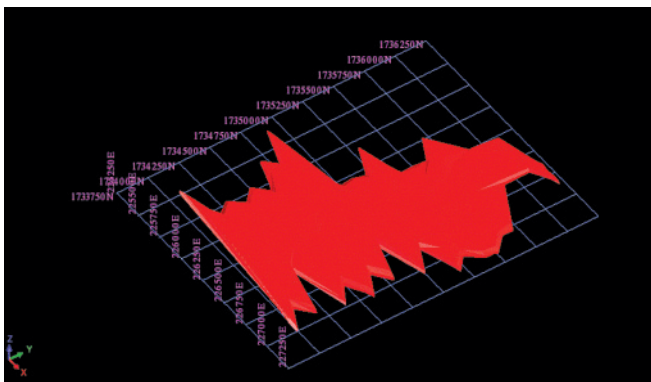| Type	Value
Number of samples	3041
Minimum value	16.81
Maximum value	50.47
Mean	45.15
Median	45.66
Geometric mean	45.03
Standard deviation	2.99
Variance	8.96
Coefficient of varriation	0.06
Skewness	-2.26
Kurtosis	15.10

Fig.5 Solid model of ore body

segments are joined one by one to form a solid orebody. At last the solid orebody is validated with no error. After validated the orebody model, the area and volume of solid report is obtained which is shown in Table.2.

2.3. BLOCK MODELLING

Block modelling is the basic need for reserve estimation process. For validation of the block model, comparison is made in between volume of block model and volume of solid

TABLE 2: OBJECT REPORT OF SOLID ORE MODELLING

Layer name : ore.dtm

Object: 8

Trisolation: 1

Validated = true

Status = solid

Trisolation extents

X minimum: 225394.714	X maximum: 227223.451
Y minimum: 1733968.000	Y maximum: 1736168.000
Z minimum: 157.183	Z maximum: 253.318

Surface area: 4544957

Volume: 87622041

TABLE 3: SUMMARY OF BLOCK MODEL

Type	Y	X	Z
Minimum coordinate	1733968	225394.714	157.183
Maximum coordinate	1736168	227234.714	157.183
User block size	20	20	10
Total blocks	42288		

ore model. For generating a block model different steps which are followed by using SURPAC software.

(a) Creation of empty block model

(b) Adding of attributes

(c) Adding of constraints

(d) Filling of the block model according to the attributes assigned

Creation of empty block model

Block model formation generally involved the block name, its perimeter and size of the block. On the basis of the bench height and width block size of 20 m × 20 m × 10 m and sub block size of 10 m × 10 m × 5 m was used for estimation.

Adding of attributes

Each block information in the block model are added in the empty blocks by attributes. They include the assay value of each mineral constituents, specific gravity, rock code, partial percentage. Value of attribute can be edited for further modification of estimation during the whole process

Adding of constraints

The selections of blocks are controlled by adding constraints in the block model. The information was retrieved from the block model by selecting the constraint for viewing the model in graphics. One or more spatial objects are logically combined for declaring a constraint. Different logical operators are employed to get the value intersected within the block model. Finally, the operation is carried out on those chosen blocks (e.g. Zhou-quan, et al., 2007).

Some of the constraints types used are: block attribute such as the grade, which is used to define the waste, high grade, low grade and medium grade ore using a set cut off value.

Filling the block model by assigned attributes

Some model attribute values are filled directly as background value which will be reflected in the block model directly. The grade value had to be assigned by interpolation method.

2.4. Block grade estimation

Inverse square distance weighting technique and ordinary kriging method are used for calculation of the block value as well as the volume and tonnage.

Inverse square distance weighting technique

Inverse distance weighting is a method which applies a weighting factor that is based on an inverse distance function to each sample with a set of known sample value about the central point of an unknown point of ore block (e.g. Stone, et al., 2004). The weighting factor is the inverse of the distance between each known sample value and the unknown central sample point, raised to the power 'n', where n is a positive integer.

where: $Z_{estimated}$ – Point of sample value to be estimated;

Z_i – Known sample value at ith location;

d_i – Distance between the point of sampled location to the point of sample location to be estimated;

n – Power of the separation distance (d) (in this paper square distance i.e. $n = 2$).

The ore block model having different colours by attribute are shown in (Fig.6). The percentage of CaO value is shown by different colours for each and every block.

Ordinary kriging

The most frequently used kriging technique is ordinary kriging. It is a geostatistical tool which is defined by the abbreviation BLUE- "best linear unbiased estimator". It is "best" because the errors in variance can be minimized, "linear" because the estimation values are linear weighted combination of the existing data, and "unbiased" since it
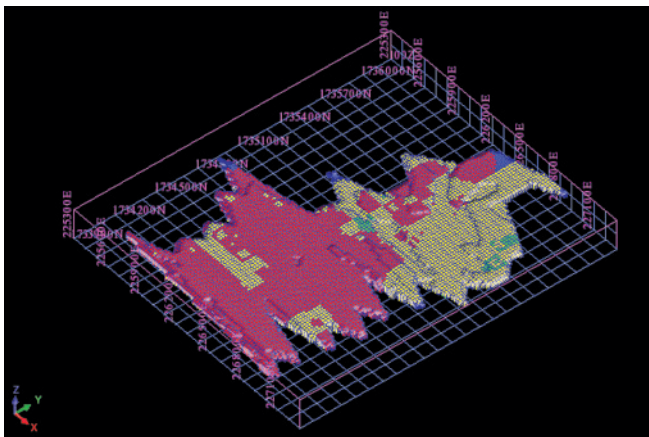

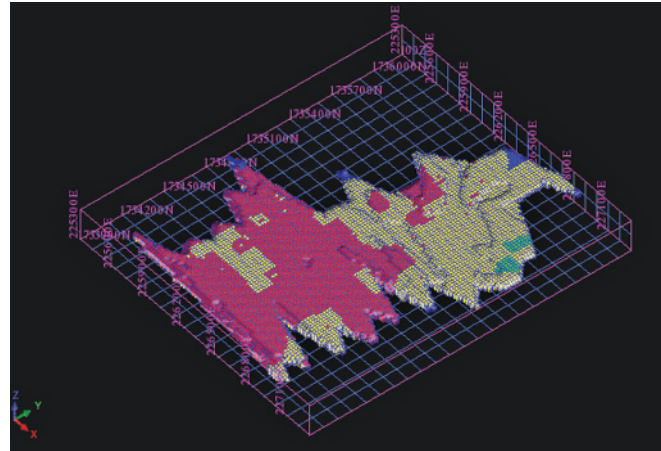Fig.6 Block model of the deposit showing colors by attribute in ISDW method

Fig.7 Block model of the deposit showing colors by attribute in OK method

attempts to have the error equal to zero (e.g. Krige, 1951).

Kriging principle is to estimate values of a delegated variable at a selected location, based on the surrounding existing values. A relevant weighting coefficient is assigned to the selected locations which represents the influence of particular data on the value of the final estimation at the selected grid node (e.g. Malvic and Balic, 2009).

The linear external parameter i.e Lagrange factor (β) is used to minimize the kriging variance in ordinary kriging method. The errors can be minimized by the limiting factor and assessment becomes impractical. The sum of all weights equal to 1, when evaluating the ordinary kriging technique (e.g. Malvic et al., 2008). The matrix form of OK equation is given below,

where – Variogram values,

$K_1, K_2, \ldots\ldots K_n$ – Known sample value at location 1, 2……. n,

X – Estimated new value location,

– Lagrange factor,

$\alpha_1, \alpha_2 \ldots\ldots \alpha_n$ – assigned weight values to the data set.

Along with different direction, variogram models are established by using the set of data. From variogram model, the values (nugget, sill and range) are used by ordinary

TABLE 4: Report of variogram model

	Model type	Nugget	Sill	Range
1	Spherical	1.11	6.55	16.94
2	Spherical	1.07	6.35	15.69

TABLE 5: Estimation result of CaO value

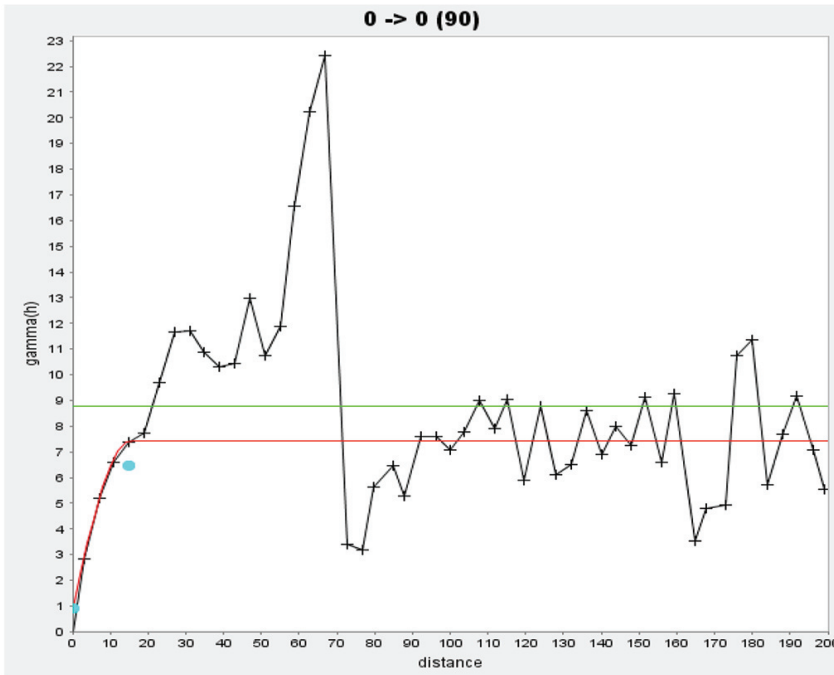| Statistical parameters	% CaO in raw data	% CaO in ISDW	% CaO in OK
Mean	45.15	45.85	44.67
Variance	8.96	12.64	21.28
Coefficient of variation	0.06	0.08	0/10
Skewness	−2.26	−4.58	−8.56

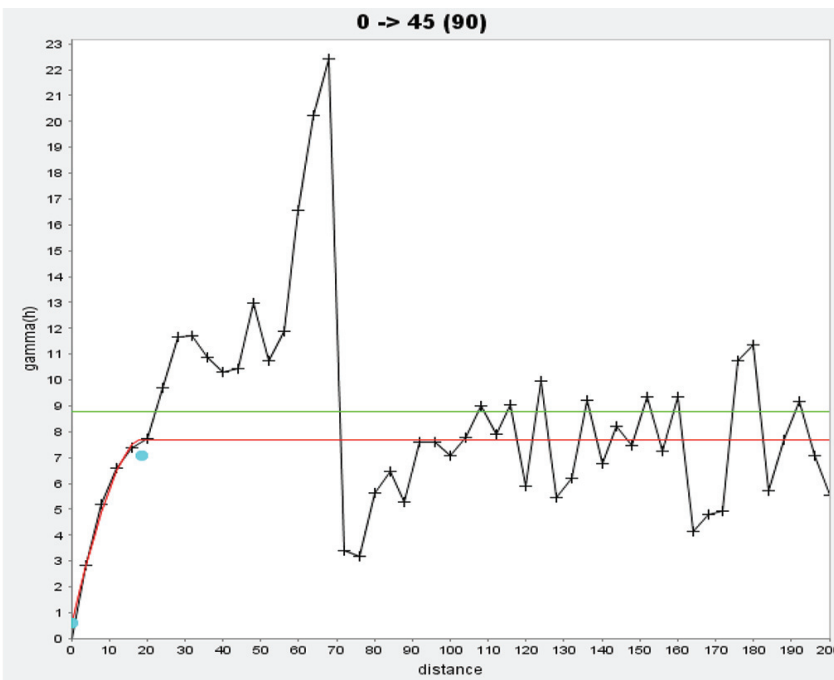Fig.8 Illustration of variogram model with azimuth 0°

Fig.9 Illustration of variogram model with azimuth 45°

kriging method. The grade value of unsampled location are estimated later on by using the calculated variogram model value. Variogram models (Figs.8-9) with azimuth 0° as well as 45° are prepared by using the present case study data.

3. Result and discussion

Conventional resource estimation is the key process to measure the resource quantitatively. This procedure is the prime estimator and time consuming process for resource valuation. In order to simplify the estimation process, the application of computer in mine planning and design is involved. In the present study the ore block model is estimated by both inverse square distance weighting and ordinary kriging methods. The estimated statistical data from both the estimator are related to the basic statistics data. From the comparison it is found that, the inverse square distance weighting method for this resource block got overemphasized. Estimated % of CaO by both ISDW and OK techniques along with the % of CaO in raw data is given below in Table 5.

The above table shows that, the estimated % of CaO value in ordinary kriging (OK) is nearer to the % of CaO value in raw data. The estimated % of CaO values in inverse square distance weighting (ISDW) overemphasizes the raw value of CaO. The estimated block grade values for different geo-statistical methods are cross checked by creating sections from the bock models. The estimated % of CaO value in ordinary kriging and % of CaO value in inverse square distance weighting are compared to % of CaO values of borehole data for the respective sections. The observation is made that, the % of CaO value in ordinary kriging is nearby to the % of CaO value in raw data. In inverse distance weighting technique, the weightage (grade value) between two sample points are taken into consideration and it takes two sample points for calculation. In kriging method, the weightage (grade value) of multiple points are taken into consideration and calculation is done by using a tedious formula, thereby giving a small average value.

4. Conclusion

From the study it is revealed that, comparing the grades of CaO for the mentioned block using the two different grade estimation techniques, found that the grades by, inverse square distance weighting and ordinary kriging methods come out to be 45.85% and 44.67% respectively. The total volume for the 3-Dimensional solid ore model is 87622041 m^3. The overall surface area of the solid model, i.e. 4544957 m^2. inverse square distance weighting method overemphasizes assay value in comparison with ordinary kriging.

5. References

1. Krige, D. G., (1951): "A statistical approach to some basic mine valuation problems on the Witwatersrand", *Journal of the Chemical, Metallurgical and Mining Society of South Africa,* Dec. 1951, pp. 119-139.

2. Isaaks, E H., Srivastava, R M., (1989): "An Introduction to Applied Geo-statistics", Oxford University Press, New York.

3. Roy, Indranil and Sarkar, B. C.: "Ore body Modelling: An Integrated Geological- Geostatistical Approach", Department of Applied Geology, IIT (ISM) Dhanbad.

4. Moharaj, M. C., and Wangmo, Y., (2014): "Ore Body Modelling and Comparison of Different Reserve Estimation Techniques" E-Thesis, National Institute of Technology, Rourkela.

5. Luo, Zhou-quan., Liu, Xiao-ming., Su, Jia-hong., Wu, Ya-bin., and Liu, Wang-ping, (2007): "Deposit 3D modelling and application", Springer publications, pp. 225-229.

6. Haldar, S. K., (2013): "Mineral Exploration- Principles and Practices", Elsevier Publications, pp.157-182.

7. Malvic, T., Cvetkovic, M. and Balic, D., (2008): "Geo-mathematical dictionary", HGD, pp.77

8. Gemcom (Tutorials)- SURPAC software.

9. Mai, Ngoc Luan., Bui, Xuan-Nam., (2016): "Application of geo-statistical tools to assess geological uncertainty for sinquyen copper mine, Vietnam", *Mining Science and Technology,* January 2016, pp. 24-29.

10. Stone, J.G., and Dunn, P.G., (1994): "Ore Reserve Estimates in the Real World", *Society of Economic Geology,* Publication No.3, pp.150-154.

11. Ramani, R.V., Mazumdar, B.K., Samaddar, A.B., (1991): "Geostatistics in Mining: The current Trend", Computers in the mineral Industry, Proceedings of the Indo-Us Seminar on Nov.11-15.

12. Malvic, T., and Balic, D., (2009): "Linearity and Lagrange Linear Multiplicator in the equation of Ordinary Kriging", Nafta, 59 (1), pp. 31-37.