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Feature Article

On the Effect of Self-Generated EM-Centrifugal Force within a Closed Circular-Loop-Electric Current and a New Type of Lamb-Shift

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Abstract
Force of electromagnetic interaction between a small element of current and the magnetic induction due to other element of current diameter-apart flowing within the same circular loop has been calculated in three different cases producing three different equations when applied to find out the expression for force-balance equations in these three different cases.

Keywords: EM Interaction, Centrifugal Force, Lorentz-Force, Bohr's Theory.

INTRODUCTION

Simple mathematical form of Laplace's equation along with Biot-Savart law and Lorentz-Force for a magnetic field caused by an electric current and the force of Electromagnetic interaction between the two current-carrying conductors comprise the basic platform of this work.

Applying the very concepts pertaining to the above-mentioned group of laws to the case of a perfectly uniform conducting wire with shape of a closed circular loop that carries a steady unidirectional electric current the force of electromagnetic interaction between any two diagonally opposite points on the loop has been calculated. It is observed that a net centrifugal force arises due to this which needs being countered. As a result elastic force within the material of the wire is supposed to be generated within it and to balance the net centrifugal force thus generated.

The consequence of all these as mentioned above is found to be surprisingly interesting, especially when extended to a domain beyond just this. This concept is then applied to the case of magnetically trapped swarm of rotating charged particles such as in Tokamak where magnetic trapping force is assumed to balance this self induced em interaction force within the circular loop. And lastly it is applied to Hydrogen atom which shows tinny shifts in the wavelengths of spectral lines of it.

THEORETICAL CALCULATION

Case (1)

Let one consider two points P and Q (as shown in Fig.1) on the circular loop carrying a unidirectional steady current '*i*'. Then due to the current-element at Q a magnetic field would be produced which at P is given by

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$$d\overrightarrow{H_P} = \frac{id\vec{l}\times(Q\vec{}P)}{4\pi\ |(Q\vec{}P)|^3} = \widehat{m}\{\frac{i\theta}{8\pi r}\} \quad \text{(i)}$$

Similarly, the magnetic field at Q due to a current element at P is the same quantity as given in Eqn.1. Now the force of em-interaction as experienced by the loop both at P and Q due to mutual influence is given by

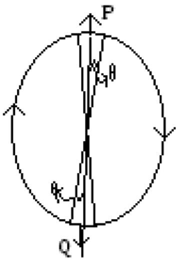

Fig.1

$$F = (\frac{\mu_0}{8\pi})i^2\theta^2 \quad \text{(ii)}$$

by magnitude (eqn.(ii)) and by direction centrifugal at both points. Here 'θ' is half of the infinitesimal angle subtended by the small element of current at the centre of the circular trajectory. Now this centrifugal force, however small it may be (say, for a current of 1 ampere and an element of angle equals to 100^{th} of a radian it is about somewhat $4.935 \times 10^{-11}N$) naturally tend to increase the radius of the loop as a whole because this amount of force acts on every point on the loop uniformly. This em-centrifugal force as a whole on the total loop length will tend to expand it by increasing either the loop-radius or for a farthest possibility by increasing the wire-diameter. This is certainly a kind of stress which immediately prompts elastic property of the wire-material to come into play. The net effective radially inward force due to this elastic restoring force within the wire-material is given by

$$F_{el} = \{\frac{Yd^2\varphi^2}{8}\} \quad \text{(iii)}$$

where 'φ' is the instantaneous increase in the angular element 'θ' of the wire considered above. Then manipulating from Eqn.(ii) with this 'φ' the balancing equation may be written as

$$F = \left(\frac{\mu_0}{8\pi}\right) i^2(\theta+\varphi)^2 = \{\frac{Yd^2\varphi^2}{8}\} \quad \text{(iv)}$$

wherefrom one gets, $\{\frac{\theta}{\varphi}\} = \sqrt{\frac{Y\pi}{\mu_0}(\frac{d}{i})} - 1$ (v)

Now for having an idea about how much can the value of this ratio approximately be, let one consider an example; For copper wire of diameter equal to 1millimeter carrying a current 1 Ampere this ratio is found out approximately to be of the order of 10^6.

Hence for a loop radius of even as big as 1 meter, the loop-length increases due to the afore-said em-interaction by only about 1μm.This is almost negligible quantity.

Case (2)

The other side of the theoretical presumption made here is its applicability to a circular-loop current comprising rotating free electrons in space. Usually electrons or other similar mutually identical charged particles in swarm are never found to revolve in a closed circular loop naturally only by themselves. An irrotational or axial magnetic field is usually known to trap charged particles in a circular loop giving rise to what is known as cyclotron machine 'Tokamak' which is generally operated to produce high-energy projectiles for Nuclear targets in the process of observing various nuclear phenomena like nuclear reactions, transmutations etc. The very simple equation for this cyclotron is given by

$$\frac{mv^2}{r} = Bev$$

where LHS of the equation represents the centrifugal force that balances the RHS representing the centripetal (Trapping) force. But now with the input from our basic consideration in the earlier section of this paper the modified equation in this case should be given by

$$\frac{mv^2}{r} + (\frac{\mu_0}{8\pi})i^2\theta^2 = Bev \quad \text{(vi)}$$

where $i = ev = (\frac{ev}{2\pi r})$ and $\theta = \frac{\Delta}{r}$, Δ being an electron's width (the mean diameter of associated electron cloud). The equation may then finally be written as $\frac{mv}{r} + \frac{\mu_0 e^2 v\Delta^2}{32\pi^3 r^4} = Be$

Now as 'Δ' is very very small compared to the ordinary cyclotron radius this part remain beyond horizon. Any cyclotron with radius comparable in magnitude with that of an electron is unrealistic and impractical.

Case (3)

In another way if the very same concept is applied in case of atomic electron by replacing the right side of the Eqn. (vi) by another relevant term one gets the following equation;

$$\frac{mv^2}{r} + \frac{\mu_0 e^2 v^2 \Delta^2}{32\pi^3 r^4} = \frac{Ze^2}{4\pi\epsilon_0 r^2} \quad \text{(vii)}$$

Here 'r' itself a very small quantity and so 'Δ' though smaller is not completely negligible as such. Bohr's theory gets modified with the inclusion of that extra magnetic term. Bohr's quantization rule and Heisenberg's uncertainty principle are considered simultaneously to reform the Eqn.(vii) in the following way; $L = mvr = \frac{nh}{2\pi}$, $\Delta p_x \Delta x \gtrsim \frac{h}{2\pi}$; Here $\Delta x = \Delta$ and $\Delta p_x = p_e$ in Eqn.(vii). For the minimum value of the uncertainty-product being equal to '$\frac{h}{2\pi}$' and therefore, $\Delta = \frac{h}{2\pi p_e}$ is taken granted. Manipulating all these values in this way to Eqn.(vii) the following equation is finally achieved :

$$\left\{\frac{Ze^2}{h^2\epsilon_0}\right\}r^2 - \left\{\frac{n^2}{\pi m}\right\}r - \left\{\frac{\mu_0 e^2}{32\pi^4 m^2}\right\} = 0 \quad \text{(viii)}$$

Rather simply written in the form of a quadratic equation as $Ar^2 - Br - C = 0$.

As A, B and C all are positive quantities and 'r' being the radius of the orbit of an electron cannot be negative the solution of the above quadratic is given by

$$r = \frac{B + \sqrt{B^2 + 4AC}}{2A} \quad \text{(ix)}$$

For evaluating the constants the following set of values of different quantities involved are taken from CODATA-2010 recommended reference [1];

e = 1.6021765658 X 10^{-19} C,

m = 0.910938291 X 10^{-30} kg,

h = 6.62606957 X 10^{-34} Js

= 4.1356675109597 X 10^{-15} eVs,

ϵ_0 = 0.8854187817 X 10^{-11} F/m,

$(\mu_0/4\pi)$ = 10^{-7} H/m .

μ_0 = 12.566370614 X 10^{-7} H/m,

π = 3.141592654,

$(1/4\pi\epsilon_0)$= 8.987551788 X 10^9m/F

C (light's speed in vaccum)= 2.99792458 X 10^8 m/s.

Putting all these values in respective formulae for the constants the values of the constants for the ground-state of Hydrogen atom are found out to be as follows;

A = 6.603284933 X 10^{39} S.I units,

B = 3.49430789 X 10^{29} S.I. units,

C = 1.2471056 X 10^{13} S.I. units.

Putting these values in Eqn.(ix) the value of Bohr-radius is found out to be r = 0.529177565Å. The value of same Bohr-radius excluding the extra magnetic term is found out to be r = 0.5291772095Å. So there is a difference of of 0.03555Fm in all which is of the order of a fraction of nuclear diameter. Again Mohr et al. [1] in their paper recommend the value of this Bohr-radius as r = 0.52917721092Å. Then the difference with the modified Bohr-radius as has been found above becomes 0.0354813015 Fm which is of the order of nuclear diameter or a little less. Here this small change is supposed to produce, though little, an appreciable change in the enrgy-spectrum because the Nuclear force is, besides being just an inverse-square type a typically higher order variational entity. Moreover for ground-state electrons' radii in case of much heavier atomic nucleus the change in the value of radius due to this additional term would be much more appreciable than this one. Another aspect of this additional term is that being a single-electron-interaction-entity the quantity is a nonquantized quantum constant for all atoms. Total energy of an orbiting electron is far more significant parameter which plays the most vital role in producing spectra.

Therefore let one investigate how much changes do occur in the wavelength of the spectral series of Hydrogen atom.

The total energy of an electron within an atom is given by

$$E_T = \frac{1}{2}mv^2 + \frac{\mu_0 i^2 \Delta^2}{8\pi r} - \frac{Ze^2}{4\pi\epsilon_0 r} = \frac{\mu_0 e^2 h^2}{256\pi^5 r^3 m^2} - \frac{Ze^2}{8\pi\epsilon_0 r} \quad \text{(x)}$$

In the formula for radius of electrons' orbit the additional part is seen to be a non-quantized quantum of energy being independent of both n and Z and being constituted of a few physical constants that yields a hundredth to one millionth of Bohr-radius amount increase in its radius. But while calculation in connection with total energy of an electron is considered this additional term, as it takes the form in equation remains no more a non-quantized one. Including ‘r’ the radius of electron’s orbit it becomes quantized. For ground state of Hydrogen atom the second term with negative sign is found out as usual to be equal to 13.6056834 eV while the additional term the first one is evaluated to be equal to 9.2μeV. Total energy of the electron is only this much amount higher than the usual one, approximately a millionth part of Rydberg unit of energy. Ordinarily this change of the order of micro-electron-volt is almost a negligible one which seems to affect the atomic energy-scheme a little. But in terms of wave-length this much amount of change in energy can produce an em wave of wavelength equal to 13.4765427cms falling in the radio-wave range. Hence the term may have a significant stand at least to some Radio-wave-applications including the radio-wave astronomy.

The radius of the orbit in general can be expressed as

$$r = (2.645886047 \times 10^{-11})\left(\frac{n^2}{Z}\right)\{1 + \sqrt{1 + (2.697745145\times10^{-6})(\frac{Z}{n^4})}\}\ \text{m} \tag{xi}$$

Putting this value in Eqn.(x) the final expression for the total energy will be

$$E = 1.176147918 \times 10^{-23}\,(\frac{Z}{n^2})^3\{1 + \sqrt{1 + (2.697745145\times10^{-6})(\frac{Z}{n^4})}\}^{(-3)} - 4.35974436\times10^{-18}(\frac{Z^2}{n^2})\{1 + \sqrt{1 + (2.697745145\times10^{-6})(\frac{Z}{n^4})}\}^{(-1)} \tag{xii}$$

Calculations for Hydrogen atom are performed and energy corresponding to different states are found out using formula (xii) and are given below;

For n = 1 (the ground state)

E = −13.6056742 eV

n = 2 (the first excited state)

E_1 = − 3.401422707 eV

n = 3 (the second excited state)

E_2 = − 1.511743591 eV

n = 4 (the third excited state)

E_3 = − 0.8503551801 eV

From the above values (calculated) of different energy-states the corresponding Hydrogen-spectral lines are found out in unit of wavelength and these are as follows:

Lyman α → 1215.024865Å,

Lyman β → 1025.176983Å,

Lyman γ → 972.019537 Å,

Balmer α → 6561.124153 Å,

Balmer β → 4860.090593 Å,

Paschen α → 18746.06563Å.

Discussion

Mohr etal [1] give their observed Lymanα line at 1216.514716Å and Lyman β line at 1026.432948Å. Parthey [2] and others [3] mentioned that Lyman α, Lyman β, Lyman γ, Balmer α and Balmer β lines are to be at 1215.7 Å, 1025.7 Å, 972.5 Å, 6562.8 Å, 4861.3 Å respectively. Draine [4] observed that spin-orbit interaction renders the Lyman α line to split into a doublet pair 1215.668Å and 1215.674Å. The small differences among observations with different single line do exist and such differences do exist also with the findings presented in this paper. But one thing is for certain that in every such case of difference the value of wavelength of respective line found here are all a little less than those reported to be observed. Parthey also observed that Deuteron charge-radius is about 1.97507 fm which is a little more than the change in the radius of the orbit of electron in the ground state of Hydrogen atom due to the additional magnetic term found here. He also observed that Lamb-shift of Lyman α related to such interaction as Paschen-Back effect is approximately 8.2GHz equivalent to an energy of amount equal to 33.9 μeV; For higher energy-level this shift will follow a (33.9μeV/n^3) rule which means that the in cases of Bracket or Pfund series the corresponding shift will be nearly equal to the shift due to the extra magnetic term the central matter of interest in this paper. Parthey [2] also mentions that change in Lyman α line frequency in terms of equivalent energy due to Zeeman effect is approximately 3-4Fev (Femtoelectronvolt). Ahmadi et al [5] mentions that multitude of absorption line at different Doppler shifted wavelengths corresponding to Lyman α line comprising 'Lyman α-forest' are used as components of a powerful tool for studying intergalactic medium and they report that the Doppler-shift in Lyman α-transition of Hydrogen with a velocity equal to $75ms^{-1}$ is found to be 0.6GHz that correspond to an energy of amount 2.48μeV approximately with an uncertainty of 0.49μeV. In this paper it is found that for ground state of Hydrogen the shift of energy from that corresponding to its normal value due to extra magnetic term is 9.2 μeV which is almost 4 times the Doppler-shift mentioned here.

Conclusion

Self-interacting-electric current-induced magnetic repulsion is the core matter of this paper. The existence of an extra force is the ultimate conclusion here. Although the force is found to be too small to be appreciated on the experimental ground in the first two cases it is not such avoidable in the third and the last case where even a tinny shift in the value of the radii of orbits of rotating electrons can produce an appreciable shift in the emission lines' frequencies. Single electron quantum system is ideal for primary studies regarding the emission and absorption spectra, and therefore, it is simpler to see how does the spectral wavelengths related to different transitions in Hydrogen atom change due to the extra magnetic term added here in this paper. As there are innumerable causes and existence of large and large number of principal emission and absorption lines, their fine-structure splits and their hyperfine-structural splits these all add to produce quite a very large span of spectrum in general. The additional term discussed here provides possibilities for some more changes in the wave lengths of different lines in that span but all in the single direction and that is a little decrease ,and not increase in every such cases.

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